Good Policy or Good Firms? International Competition and Aggregate Growth in a Granular World

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Abstract

This paper theoretically and quantitatively evaluates the hypothesis that, due to the existence of large firms (granularity), idiosyncratic shocks to individual firms can lead to significant variation in the growth of countries. I embed granularity, through finiteness in the set of firms, in a general equilibrium environment featuring monopolistic competition, growth, and international trade. Firm productivities grow according to idiosyncratic productivity shocks, which obey a Gibrat's law proportional growth process, and are the only source of growth in the model. I derive an approximate analytic mapping for the standard deviation of GDP growth in this framework, which is non-zero due to granularity. This mapping depends on only a few key parameters, which I estimate for a wide-range of countries using firm-level micro data. My results indicate that idiosyncratic shocks to firms can play a significant role in generating both short-run macroeconomic fluctuations and variation in longer-run growth trends, particularly for countries that engage heavily in international trade. Empirically, I show that the model does well in matching relative differences in GDP volatility across OECD countries.

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I. Introduction

Why do countries grow more in some periods than others? Easterly, Kremer, Pritchett, and Summers (1993) show that despite relatively high stability in country characteristics and policy across decades, cross-decade growth is highly unstable. They argue that variation in the growth of countries, therefore, appears to be driven primarily by aggregate shocks, or luck, rather than policy. Cochrane (1994) shows, however, that along with policy shocks, well defined aggregate shocks often have difficulty explaining short-run macroeconomic fluctuations. These results raise the question of what drives variation in the growth of countries. In a recent paper, Gabaix (2011) advances the hypothesis that short-run macroeconomic fluctuations can arise directly from idiosyncratic shocks to large firms. This "granular" hypothesis indicates a break from the view that shocks to individual firms wash out at the aggregate level, due to the large number of firms, and relies on the observation that for many countries a small portion of firms account for a large fraction of overall output. In this paper, I extend the granular hypothesis, proposing that idiosyncratic shocks to firms can play a significant role in not only generating short run macroeconomic fluctuations, but also variation in longer-run growth trends. Further, I argue that the magnitude of aggregate variation that arises due to idiosyncratic shocks increases as countries engage more heavily in international trade.

Theoretically and quantitatively evaluating this extended granular hypothesis necessitates a break from frameworks in which all firms are infinitesimal relative to the size of an economy, to ones in which individual firms have positive mass and are therefore able to affect macroeconomic aggregates. I embed granularity, through finiteness in the set of firms, in a general equilibrium framework featuring growth, monopolistic competition among firms, and international trade. The only source of growth in the framework is due to idiosyncratic shocks to the productivities of individual firms, which obey a Gibrat's law proportional growth process. Section III specifies the full environment and derives an approximate analytic mapping for the standard deviation of constant price GDP growth, which is non-zero due to granularity. This mapping depends on only a few key parameters, which are straightforward to estimate,

and highlight the forces governing the degree to which aggregate variation arises from idiosyncratic shocks to firms in this environment. The primary parameter governing this mapping is how concentrated output is among firms, or how granular the economy is, as measured by the herfindahl index of the economy. The other parameters that govern this mapping are the growth process for firms (in particular the volatility of firm growth), how substitutable output is across firms, and the degree to which a country's largest firms compete internationally versus domestically.

International competition, relative to domestic competition, amplifies the impact individual firms can have on an economy, as when a firm increases its output, its competitors react by lowering their output. If these competitors are domestic firms, this partially negates the aggregate impact of the initial increase. When there is international competition, however, some of the competitors reacting are located outside of the country. When market shares are exchanged internationally, this has a larger aggregate impact than when market shares are reshuffled among firms domestically. The intuition is that for the U.S. economy, the decline of Sears has been largely offset by, and, in fact, has contributed to, the rise of Walmart and Target. For Finland's economy, however, the decline of Nokia has larger implications, even conditioning on Nokia's size relative to Finland's economy, as Nokia's lost sales are not going to other firms in Finland, but rather to firms outside of the country such as Samsung and Apple. International competition, therefore, amplifies the aggregate variation that can arise due to idiosyncratic shocks to firms.

Whereas previous research has focused on the role of granularity in generating short-run fluctuations (quarterly or annual), this paper extends the granular framework to examine the extent to which idiosyncratic shocks to firms can lead to variation in aggregate growth over longer periods of time, focusing especially on variation in decade-to-decade growth. Due to the Gibrat's law growth process for firm productivities, the effects of idiosyncratic shocks accumulate over time leading to higher variation in the growth of firms and therefore higher variation in GDP growth over longer periods of time. This feature is consistent with the fact that the standard deviation of 10-year firm

growth rates is significantly higher than the standard deviation of 1-year firm growth rates, and similarly for 10-year GDP growth rates compared to 1-year GDP growth rates.

In section V, I estimate the parameters of the framework for a wide-range of countries using aggregate data on exports, expenditures, and price indices, as well as firm-level data on sales and gross-margins. I plug these parameter estimates into the analytic mapping derived in section III to quantitatively evaluate the question: "How much variation in GDP growth would we expect in each country if differences in the realizations of idiosyncratic shocks to firms were the only source of aggregate variation?" My quantitative results indicate that idiosyncratic shocks to firms, of the order estimated in firm-level data, can be a significant source of aggregate variation, particularly for countries that engage heavily in international trade. For OECD countries, the model predicts a standard deviation of both 1-year and 10-year GDP growth rates roughly half the magnitude observed in the data. These results suggest that idiosyncratic shocks to firms can potentially play a large role in not only explaining why countries grow more in some years than others, as argued by Gabaix (2011), but also why countries grow more in some decades than others. The granular hypothesis, therefore, provides a potential microfoundation for the aggregate "luck" shocks of Eaton et. al. (1993) in terms of accumulated idiosyncratic productivity shocks to the largest and fastest growing firms in an economy.

In section VI, I show that the predictions of the model are consistent with several stylized facts regarding cross-country differences in GDP growth volatility. In particular, I find that the model does well in matching relative differences in the observed volatility of annual GDP growth rates across OECD countries (R-squared of 0.47), indicating the likely importance of idiosyncratic shocks in generating aggregate fluctuations. Consistent with the predictions of the model, I find that countries which export more as a share of GDP exhibit a higher volatility of annual GDP growth rates, and similarly for countries that are more concentrated in their output as measured by their Herfindahl index. After establishing that the model does well in matching stylized facts regarding annual GDP growth rate volatility, I focus on the model's predictions regarding cross-

country variation in growth. I group countries together depending on their predicted variation of 10-year GDP growth rates in the model and show that countries in groups with higher predicted variation in 10-year GDP growth rates do indeed exhibit higher cross-country variation in their growth over 1995–2005, with the model again generating roughly half the variation observed in the data.

II. Related Literature

This paper builds on recent research regarding the potential microeconomic origins of aggregate variation. Gabaix (2011) and Carvalho and Gabaix (2013) explore the extent to which idiosyncratic shocks to firms can explain U.S. GDP volatility in granular environments, focusing particularly on the role of linkages across firms and sectors. Carvalho and Graiss (2015) embed granularity in a dynamic Hopenhayn (1992) framework to study how firm dynamics can drive business cycles. Notably, these papers do not require frictions in order to generate aggregate variation from idiosyncratic shocks to firms; instead it arises naturally due to high concentration of output among firms of the magnitude observed in the data. This contrasts with non-granular environments, such as Arellano, Bai, and Kehoe (2012), in which idiosyncratic shocks to firms affect aggregate output only under the presence of financial frictions. A parallel strand of research explored by Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012), Horvath (2000), Dupor (1999), Ataly (2014), Koren and Tenreyro (2007), and Caplin and Leahy (1993) focuses instead on sectoral granularity and the role of sectoral shocks in generating aggregate fluctuations.

This paper relates to the literature on Gibrat's law based models of firm growth, recent examples of which are papers by Luttmer (2007, 2010) and Arkolakis (2013), by showing how significant aggregate variation arises in these models when there are a finite number of firms. In the models of both Luttmer and Arkolakis, the underlying Gibrat's law based productivity ultimately leads to a right tail that follows a Pareto distribution. Gabaix (2011) notes that when the distribution of firms sizes follows a Pareto distribution, output will remain highly concentrated among firms even when

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there is a large number of firms overall. Di Giovanni and Levchenko (2012) make use of the insight from Melitz (2003), that lowering barriers to international trade causes small firms to exit and large firms to become larger, to show how international trade increases the concentration of output among firms and therefore increases volatility in granular economies, particularly for smaller countries.

III. Theoretical Framework

This section lays out the framework of the model. There are two necessary elements required to generate non-zero aggregate variation due to variation in the growth of firms. These elements are finiteness in the set of firms, which Gabaix (2011) refers to as granularity, and idiosyncratic variation in the growth process of firms. To understand how these features translate to aggregate variation in an environment where shocks to firms affect other firms in the economy, I embed these features in an otherwise standard general equilibrium framework featuring international trade and multiple industries, where heterogeneous firms engage in monopolistic competition within each industry ala Krugman (1979) and Dixit and Stiglitz (1977).

III.A. A Simple Example

Before describing the general equilibrium environment, I present a simple example based on the framework of Gabaix (2011) showing how idiosyncratic shocks to firms can generate aggregate variation due to granularity in an environment where shocks to output are independent across firms. Suppose a firm accounts for 10% of a country's aggregate output, and receives a shock that increases its output by 15%. This shock will therefore increase aggregate output by 1.5% ($= 10\% * 15\%$). Similarly, the variance in aggregate output growth due to this firm will be $(10\%)^2 \sigma_{firm}^2$, where σ_{firm}^2 is the variance of the firm's shocks. Assuming that shocks are independent across firms, the variance of aggregate output resulting from shocks to individual firm shocks will be $\sigma_{Agg}^2 = \sum_{m=1}^{M}(s_m)^2 \sigma_m^2$ where s_m is firm m's share of total output and σ_m^2 is the variance of firm m's shock. When this variance is identical across firms, the formula simplifies to $\sigma_{Agg}^2 = \sigma_{firms}^2 h$, where $h = \sum_{m=1}^{M}(s_m)^2$ is the Herfindahl index for this set of firms.

Therefore in this simple framework, aggregate variation is inherited directly from variation in firm shocks, scaled by a measure of the concentration of output among firms.

III.B. General Equilibrium Framework

There are $i, j = 1, \ldots, N$ countries, and $k = 1, \ldots, K$ industries, where K^{TR} of the industries are tradable and K^{NT} are non-tradable industries. As an abuse of notation, I also let K^{TR} and K^{NT} denote the set of traded and non-traded industries respectively. There is measure L_i of consumers in country i, each with 1 unit of labor, which is supplied inelastically. Consumers have Cobb-Douglas preferences over industries, with period utility at time t given by:

$$U_{i,t} = \sum_{k=1}^{K} \theta_k \log C_{i,k,t},$$

where $C_{i,k,t}$ is consumption of industry $k's$ final output and θ_k is industry $k's$ expenditure share. Consumers earn income from wages and profits from firms, which are rebated to the consumers in the country the firm is headquartered in. Country $i's$ expenditures on industry k are therefore given by $E_{i,k,t} = \theta_k(w_{i,t}L_i + \pi_{i,t})$, where $\pi_{i,t}$ is the total profit of country $i's$ firms and $w_{i,t}$ is the wage of country i.

Industries can be either traded or nontraded. In non-traded industries, market clearing ensures that $C_{i,k,t} = Y_{i,k,t}^i$, where $Y_{i,k,t}^i$ is domestic industry output. In traded industries, industry consumption is a CES bundle of foreign and domestic industry output. Perfectly competitive bundlers solve

$$\min \sum_{j=1}^{N} P_{j,k,t}^i Y_{j,k,t}^i$$

subject to their technology for creating final consumption output in industry k.

$$Y_{i,k,t}^C := \left(\sum_{j=1}^{N} (Y_{j,k,t}^i)^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}$$

Within each industry, there are a finite number of firms each producing a differentiated varieties. Differentiated varieties are transformed into industry output using a constant elasticity of substitution (CES) production function by perfectly competitive set of output bundlers in each industry. Final output producers in country i for industry k minimize costs by selecting output from the $m = 1, \dots, M_{i,k}$ differentiated firms within each industry:

$$\min \sum_{m=1}^{M_{i,k}} y_{i,k,t}^{j,m} y_{i,k,t}^{j,m}$$

subject to the production function

$$Y_{i,k,t}^{j} = \left(\sum_{m=1}^{M_{i,k}} (y_{i,k,t}^{j,m})^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}$$

Here ϵ is the elasticity of substitution between products within an industry, $y_{i,k,t}^{j,m}$ is the output of the variety produced by firm m located in country i used in country j at time t, and $p_{i,k,t}^{j,m}$ is the price of this output. When the industry is non-traded, $y_{i,k,t}^{j,m} \equiv 0$ for $j \neq i$, as firms are not able to export non-traded varieties to foreign countries.

Firms differ in their productivity and produce their variety using labor, which is supplied inelastically. As in Atkeson and Burstein (2008), there are a finite number of firms in each industry, although here there are a finite number of industries as well. I assume there are no trade costs in traded sectors, therefore output of firm m produced for country j is be given by:

$$y_{i,k,t}^{j,m} = z_{i,k,t}^{m} l_{i,k,t}^{j,m},$$

where $z_{i,k,t}^{m}$ is firm m's productivity, $l_{i,k,t}^{j,m}$ is the amount of labor used to produce output for country j. The optimization of industry bundlers yields the quantity demanded of each variety as a function of prices as:

$$y_{j,k,t}^{i,m} = \frac{E_{i,k,t}}{\left(p_{j,k,t}^{i,m}\right)^{\frac{1}{\epsilon}} \left(P_{i,k,t}^{C}\right)^{\epsilon-1}},$$

where $P^C_{i,k,t}$ is the overall price index for industry consumption in country i

$$P^C_{i,k,t} = \left(\sum_{j=1}^{N} \sum_{m=1}^{M_{i,k}} \left(p^{i,m}_{j,k,t} \right)^{\epsilon-1} \right)^{\frac{1}{\epsilon-1}}.$$

A common assumption within this CES framework is that firms either have no effect on (as is the case when there is a continuum of firms), or ignore their effect on, total industry output. When firms face linear output costs this results in firms charging a constant markup over marginal cost. In a granular framework, keeping this assumption implies that, although firms affect the industry price index, they do not take this into account when optimizing. There are two reasons for imposing this assumption. The primary reason is that this paper is about how having a finite number of firms leads to variation in aggregate variables, and to make the results as comparable as possible to a non-granular framework I hold fixed the pricing behavior of large firms across the granular and non-granular frameworks. The second reason is more practical: this assumption allows me to solve for equilibrium analytically. This assumption can be discarded, and the equilibrium can still be solved numerically. Di Giovanni and Levchenko (2012), however, show that imposing this restriction in unlikely to significantly affect the magnitude of aggregate variation. Still, ignoring the effects of changes in markups is somewhat at odds with the large literature dealing with how markups are affected by international trade, for instance Locker, Goldberg, Khandelwal, and Pavcnik (2012) and Edmond, Midrigan, and Xu (2012). An alternative possibility would be to relax the assumption of constant markups without allowing full Cournot or Bertrand competition across firms. For example, the equilibrium could still be solved analytically if we allow for endogenous markups resulting from either non-linear output costs as in Hottman, Redding, and Weinstein (2014) or misallocation as in Peters (2013). Similarly, it's possible that models that deviate from the standard CES framework, for example through adding non-homotheticities to the CES production functions as in Sato (1977), or to consumer preferences as in Simonovska (2010), may increase the importance of changes in markups.

Under the assumption that firms optimize without taking into account their effect on the industry price index, the firm's price will be a constant markup over its marginal cost:

$$p_{i,k,t}^m = \left(\frac{\epsilon}{\epsilon - 1}\right) \frac{w_{i,t}}{z_{i,k,t}^m},$$

where the markups depend on the elasticity of substitution across differentiated products within an industry. In this constant markup framework, the profits earned by a firm are proportional to the amount of inputs used by that firm:

$$\pi_{i,k,t}^m = \left(\frac{1}{\epsilon - 1}\right) \sum_{j=1}^N l_{i,k,t}^{j,m},$$

and firm sales are proportional to the firm's productivity raised to the power $\epsilon - 1$:

$$p_{i,k,t}^m y_{i,k,t}^m = \sum_{j=1}^N p_{i,k,t}^{j,m} y_{i,k,t}^{j,m} = \left(\frac{z_{i,k,t}^m}{w_{i,k,t}}\right)^{\epsilon-1} \frac{\theta_k \left(\frac{\epsilon-1}{\epsilon}\right) \sum_{j=1}^N L_j}{\sum_{j=1}^N \sum_{m'=1}^{M_{j,k}} \left(\frac{z_{j,k,t}^{m'}}{w_{j,t}}\right)^{\epsilon-1}}.$$

This last equation indicates that when varieties are highly substitutable, a small increase in productivity leads to a large increase in sales.

III.C Linking Aggregate Growth to Firm Growth

Starting at an initial time $t_0 = 0$, I compute the constant price GDP growth of country i after T periods as:

$$\Delta_{i,GDP,T} := \frac{\sum_{k=1}^K P_{i,k,0} Y_{i,k,T}}{\sum_{k=1}^K P_{i,k,0} Y_{i,k,0}} = \frac{\sum_{k=1}^K P_{i,k,0} Y_{i,k,0} \left(\frac{Y_{i,k,T}}{Y_{i,k,0}}\right)}{\sum_{k=1}^K P_{i,k,0} Y_{i,k,0}}$$

Where $P_{i,k,0}$ is the initial price index of country i's output, and $Y_{i,k,t} = \sum_{j=1}^N Y_{i,k,t}^j$ is country i's total output in industry k at time t.

III.C.i Linking Aggregate Growth to Firm Growth: Autarky

If the country is in autarky, then $P_{i,k,0} Y_{i,k,0} = \theta_k E_{i,k,0}$, which means that constant price GDP growth can be expressed as

$$\Delta_{i,GDP,T}^{aut} = \sum_{k=1}^{K} \theta_k \frac{Y_{i,k,T}}{Y_{i,k,0}} = \sum_{k=1}^{K} \theta_k \left(\sum_{m=1}^{M_{i,k}} \frac{\left(z_{i,k,t}^m\right)^{\epsilon-1}}{\left(\sum_{m=1}^{M_{i,k}}\left(z_{i,k,0}^m\right)^{\epsilon-1}\right)} \right)^{\frac{1}{\epsilon-1}}.$$

Now let $X_{i,k,T}^m = z_{i,k,T}^m / z_{i,k,0}^m$ be firm m's productivity growth after T periods, then in autarky, we can write constant price GDP growth as a weighted power mean of productivity shocks to firms in each industry where the power is $\epsilon - 1$ and each firm's productivity shock is weighted by its share of industry sales for firms located in that country:

$$\Delta_{i,GDP,T}^{aut} = \sum_{k=1}^{K} \left(\sum_{m=1}^{M_{i,k}} \left(X_{i,k,T}^m\right)^{\epsilon-1} \left(\omega_{i,k,0}^{m,aut}\right) \right)^{\frac{1}{\epsilon-1}}$$

where $\omega_{i,k,0}^{m,aut}$ is the weight for each firm in autarky

$$\omega_{i,k,0}^{m,aut} := \theta_k \frac{p_{i,k,0}^m y_{i,k,0}^m}{\sum_{m=1}^{M_{i,k}} p_{i,k,0}^m y_{i,k,0}^m} = \theta_k \frac{\left(z_{i,k,0}^m\right)^{\epsilon-1}}{\sum_{m=1}^{M_{i,k}} \left(z_{i,k,0}^m\right)^{\epsilon-1}}$$

and note that $\sum_{k=1}^{K} \sum_{m=1}^{M_{i,k}} \omega_{i,k,0}^{m,aut} = 1$.

III.C.ii Linking Aggregate Growth to Firm Growth: Trade

When a country engages in trade, expenditures do not necessarily equal production in traded sectors. Instead we have the balanced trade condition:

$$\sum_{k=1}^{K^{TR}} \sum_{\substack{j=1 \\ j \neq i}}^{N} P_{i,k,t} Y_{i,k,t}^j = \sum_{k=1}^{K^{TR}} \sum_{\substack{j=1 \\ j \neq i}}^{N} P_{j,k,t} Y_{j,k,t}^i$$

while total industry expenditures across countries still equals total industry production:

$$\sum_{j=1}^{N} P_{j,k,t} Y_{j,k,t} = \theta_k \sum_{j=1}^{N} E_{j,t}.$$

In the case with trade, constant price GDP growth simplifies to (I normalize $w_{i,t} = 1$)

$$\Delta_{i,GDP,T} = \sum_{k=1}^{K^{NT}} \theta_k \left(\sum_{m=1}^{M_{i,k}} \frac{\left(z_{i,k,t}^m\right)^{\epsilon-1}}{\left(\sum_{m=1}^{M_{i,k}} \left(z_{i,k,0}^m\right)^{\epsilon-1}\right)} \right)^{\frac{1}{\epsilon-1}}$$

$$+ \left(\sum_{k=1}^{K^{TR}} \theta_k \right) \left(\frac{\sum_{j=1}^N E_{j,t}}{\sum_{j=1}^N E_{j,0}} \right) \sum_{k=1}^{K^{TR}} \frac{\theta_k \left(\sum_{m=1}^{M_{i,k}} s_{i,k,t}^m \right)}{\sum_{k=1}^{K^{TR}} \theta_k \sum_{m=1}^{M_{i,k}} s_{i,k,0}^m} \left(\sum_{m=1}^{M_{i,k}} \frac{\left(z_{i,k,t}^m\right)^{\epsilon-1}}{\left(\sum_{m=1}^{M_{i,k}} \left(z_{i,k,0}^m\right)^{\epsilon-1}\right)} \right)^{\frac{1}{\epsilon-1}}$$

where $s_{i,k,T}^m$ is firm m's share of sales in industry k for country i in period t:

$$s_{i,k,t}^m := \frac{p_{i,k,t}^m y_{i,k,t}^m}{\sum_{j=1}^N \sum_{m'=1}^{M_{i,k}} p_{j,k,t}^{m'} y_{j,k,t}^{m'}} = \frac{\left(z_{i,k,t}^m\right)^{\epsilon-1}}{\sum_{j=1}^N \sum_{m=1}^{M_{i,k}} \left(\frac{z_{j,k,t}^m}{w_{j,t}}\right)^{\epsilon-1}}. \qquad (1)$$

Again let $X_{i,k,T}^m = z_{i,k,T}^m / z_{i,k,0}^m$ be firm m's productivity growth after T periods, then the first term of $\Delta_{i,GDP,T}$, which represents growth in nontraded industries, will be a weighted power mean as in the case of autarky. The second term of $\Delta_{i,GDP,T}$, representing growth in the traded sector, is again a weighted power mean, as

$$\sum_{k=1}^{K^{TR}} \sum_{m=1}^{M_{i,k}} \frac{\theta_k \left(\sum_{j=1}^N E_{j,t}\right) \left(s_{i,k,t}^m\right)}{\sum_{k=1}^{K^{TR}} \theta_k \left(\sum_{j=1}^N E_{j,0}\right) \left(\sum_{m=1}^{M_{i,k}} s_{i,k,0}^m\right)} = 1,$$

however, unlike for non-traded industries, the weights are no longer independent of the realizations of productivity shocks. Therefore we can write

$$\Delta_{i,GDP,T} = \sum_{k=1}^{K} \left(\sum_{m=1}^{M_{i,k}} \left(X_{i,k,T}^m\right)^{\epsilon-1} \omega_{i,k,T}^m \right)^{\frac{1}{\epsilon-1}}, \qquad (2)$$

where

$$\omega_{i,k,T}^m = \begin{cases} \theta_k \dfrac{\left(z_{i,k,0}^m\right)^{\epsilon-1}}{\sum_{m=1}^{M_{i,k}} \left(z_{i,k,0}^m\right)^{\epsilon-1}}, & k \in K^{NT} \\[2em] \left(\left(\sum_{k=1}^{K^{TR}} \theta_k \right) \dfrac{\theta_k \left(\sum_{j=1}^N E_{j,t}\right) \left(\sum_{m=1}^{M_{i,k}} s_{i,k,T}^m\right)}{\sum_{k=1}^{K^{TR}} \theta_k \left(\sum_{j=1}^N E_{j,0}\right) \sum_{m=1}^{M_{i,k}} s_{i,k,0}^m} \right)^{\epsilon-1} s_{i,k,0}^m, & k \in K^{TR} \end{cases} \qquad (3)$$

and again $\sum_{k=1}^K \sum_{m=1}^{M_{i,k}} \omega_{i,k,T}^m = 1$.

This formula is very similar to the autarky case, except now, for traded sectors, the weight for each firm's productivity growth is no longer independent of the productivity growth of firms in the economy. The reason for this, is that international trade allows labor to move from less productive firms in one traded industry to more productive firms in other traded industries. Without trade, constant expenditure shares ensure that a constant fraction of labor is allocated to each sector, regardless of the productivity of the sector. Trade allows countries to specialize in producing the industry varieties that they are more productive in, while importing the industry varieties that they are less productive in. This will influence both the initial Herfindahl of the economy, as countries will specialize in the industries they are most productive in – which is determined by how productive the firms in the industry are.

III.D Productivity Growth Process for Firms

I assume that that the underlying productivity growth process is common across firms, while the realizations of the productivity growth process are idiosyncratic for each firm. This is a form of Gibrat's Law, which says that the growth rate of each firm is independent of the size of the firm. Productivity growth evolves according to a geometric Brownian motion process:

$$dX_T = \mu X_t dT + \sigma X_T dW_T, \qquad (4)$$

$$X_0 = 1,$$

where μ is the drift (average annual growth rate of logged firm productivity), and σ is the volatility (standard deviation of logged firm productivity growth), and W_T is a standard Wiener process. The realizations of the Wiener process are unique across firms. This process implies that productivity growth follows a lognormal distribution with mean and variance after T periods given by:

$$E[X_T] = e^{\mu T}, \qquad (5)$$

$$Var[X_T] = e^{2T\mu}\left(e^{T\sigma^2} - 1\right). \qquad (6)$$

Empirical research has repeatedly shown that firm growth is well approximated by geometric Brownian motion (GBM), for example see Cabral and Mata (2003),

Arkolakis (2013), Singh and Whittington (1975) among many others. Similarly, GBM growth processes are capable of generating many of the stable firm size distributions proposed to fit observed firm sizes. For instance, Stanley et. al. (1995) claims that firm sizes follow a lognormal distribution, which can be generated by a pure GBM process on firm productivity. Axtell (2001) argues that firm sizes are instead best approximated by a Pareto distribution, which can be achieved by adding a lower reflective bound for productivity leads to a Pareto distribution as shown in Gabaix (1999). Other possibilities that can be generated through modifications to an underlying GBM process are a double Pareto distribution as in Arkolakis (2013), or for logged size to follow a mixture of gamma distributions as in Luttmer (2007). I refer the reader to Luttmer (2010) and de Wit (2005) for surveys of the conditions under which GBM produces various distributions. As there is still some controversy and disagreement over what the distribution of firm sizes is in the data, a strength of this framework is that it is not necessary to take a stand on what the initial distribution of firm sizes "should be" or how the initial distribution of firm productivities arises. Instead the framework is compatible with an arbitrary distribution of firm productivities, which can be inferred from firm-level data.

III.E Deriving Variation in GDP Growth

In this section I derive an expression for the standard deviation of constant price GDP growth after T periods, conditional on an arbitrary initial distribution of firm productivities, and the productivity growth process for firms. The distribution of GDP growth is a weighted power mean of lognormal random variables. No simple expression for the density of the sum of lognormal random is known, and, except in the case of autarky where $\epsilon = 2$ leading to linearity in equation (2), there is no known closed-form expression for the first two moments of the weighted power mean of lognormal random variables. I instead focus on deriving a simple approximate analytic expression for the mean and standard deviation of GDP growth. There are two benefits to having an analytic expression as opposed to simulating the model. The first is that it makes it clear what role every part of the framework plays in generating and amplifying variation in

GDP growth. The second is that is that it makes it easy to quantitatively evaluate the model and apply it across countries, as only a few parameters will need to be estimated for each country.

III.E.i Deriving Variation in GDP Growth: Autarky

In the case of autarky, the weights of the weighted power mean in equation (2) are constant and do not change depending on the shock. An exact closed form expression for the distribution of GDP growth cannot be given, however, an approximation for the mean and variance of GDP growth can be found through a double application of the delta method which makes use of taylor approximations. The approximation yields:

$$E[\Delta_{i,GDP,T}^{aut}] \approx \xi(T,\sigma,\epsilon)E[X_T], \qquad (7)$$

$$Var[\Delta_{i,GDP,T}^{aut}] \approx (\xi(T,\sigma,\epsilon))^2 h_{i,0}^{aut} Var[X_T], \qquad (8)$$

where $E[X_T]$ and $Var[X_T]$ are from (5) and (6), and σ^2 is from the GBM process in (4),

$$h_{i,0}^{aut} := \sum_{k=1}^{K} \sum_{m=1}^{M_{i,k}} \frac{(p_{i,k,t}^m y_{i,k,t}^m)^2}{\left(\sum_{k=1}^{K} \sum_{m=1}^{M_{i,k}} p_{i,k,t}^m y_{i,k,t}^m\right)^2} = \sum_{k=1}^{K} (\theta_k)^2 \sum_{m=1}^{M_{i,k}} \left(\frac{(z_{i,k,0}^m)^{\epsilon-1}}{\sum_{m'=1}^{M_{i,k}} (z_{i,k,0}^{m'})^{\epsilon-1}}\right)^2, \qquad (9)$$

is the sales Herfindahl of the economy in autarky and

$$\xi(T,\sigma,\epsilon) = \left(1 + \frac{(\epsilon-1)(\epsilon-2)}{2}\left(e^{T\sigma^2} - 1\right)\right)^{\frac{1}{\epsilon-1}}, \qquad (10)$$

is a term that captures the expected additional reallocation that results from non-linearity in the power mean (for $\epsilon \geq 2$). The behavior of this term is illustrated in figure 1. The term arises since, as the elasticity of substitution increases, firms that receive high productivity shocks will increase their labor input more, and due to this additional reallocation, constant price GDP grows slightly more than if output were less substitutable across firms. An important result of this analytic mapping is that the sales Herfindahl is a sufficient statistic for how the distribution of initial firm productivities effects the standard deviation of GDP growth. As mentioned previously, this makes it

significantly easier to apply the model to make predictions, as we will not require the full distribution of firm sizes, nor even the Herfindahl for each industry.

III.E.i Deriving Variation in GDP Growth: Trade

In the case of trade, weights for firms in the traded sector are no longer independent from the shocks firms receive, as countries are able to specialize in the industries in which their relatively most productive firms reside. To assist in deriving an approximate mapping for GDP growth in this case I make two simplifying assumptions. First, I assume that there is no uncertainty in aggregate growth for the combined rest of world. This is a limiting case of assuming that the rest of the world is large enough that output is not highly concentrated among firms at a global level. The second assumption is that there is a large number of traded industries with a large number of firms in each industry. The accuracy of the approximate mapping decreases as the number of traded industries decrease, as it under predicts the dampening effects of general equilibrium changes in relative wages. For example, in the case where there is only a single traded industry, trade does not amplify variation in GDP growth at all. With these assumptions imposed, the approximate mapping for GDP growth is derived in a similar way to the autarky with a double application of the delta method. The approximate mapping yields, for $\epsilon \geq 2$:

$$E\left[\Delta_{i,GDP,T}\right] \approx \xi(T,\sigma,\epsilon)E[X_T], \qquad (11)$$

$$Var\left[\Delta_{i,GDP,T}\right] \approx \left(\psi\left(\epsilon, S_{i,0}\right)\right)^2 \left(\xi(T,\sigma,\epsilon)\right)^2 Var[X_T]h_{i,0}, \qquad (12)$$

where again $E[X_T]$ and $Var[X_T]$ are from (5) and (6), σ^2 is from the GBM process in (4),

$$h_{i,0} := \sum_{k=1}^{K}\sum_{m=1}^{M_{i,k}} \frac{\left(p_{i,k,t}^m y_{i,k,t}^m\right)^2}{\left(\sum_{k=1}^{K}\sum_{m=1}^{M_{i,k}} p_{i,k,t}^m y_{i,k,t}^m\right)^2}$$

$$= \sum_{k=1}^{K^{NT}} (\theta_k)^2 \sum_{m=1}^{M_{i,k}} \left(\frac{(z_{i,k,t}^m)^{\sigma_k - 1}}{\sum_{m=1}^{M_{i,k,t}} (z_{i,k,t}^m)^{\sigma_k - 1}} \right)^2 + \left(\sum_{k=1}^{K^{TR}} \theta_k \right)^2 \sum_{k=1}^{K^{TR}} \sum_{m=1}^{M_{i,k}} \frac{(\theta_k)^2 (s_{i,k,t}^m)^2}{\left(\sum_{k=1}^{K^{TR}} (\theta_k) \sum_{m=1}^{M_{i,k}} (s_{i,k,t}^m) \right)^2}, \qquad (13)$$

is the sales Herfindahl of the economy, which although defined the same as in autarky, will differ due to specialization in productive traded industries ($h_{i,0} \geq h_{i,0}^{aut}$). We again have $\xi(T, \sigma, \epsilon)$ as defined in equation (10),

$$\xi(T, \sigma, \epsilon) = \left(1 + \frac{(\epsilon - 1)(\epsilon - 2)}{2} (e^{T\sigma^2} - 1) \right)^{\frac{1}{\epsilon - 1}},$$

and $\psi(\epsilon, S_{i,0})$ captures the additional amplifying effect of international trade outside of its effect on the Herfindahl index of an economy,

$$\psi(\epsilon, S_{i,0}) = \frac{(1 + (\epsilon - 1)S_{i,0})}{(1 + S_{i,0})}, \qquad (14)$$

where $S_{i,0}$ is the country's initial exports as a share of GDP,

$$S_{i,0} := \frac{\sum_{\substack{j=1 \\ j \neq i}}^{N} \left(\sum_{k=1}^{K} \sum_{m=1}^{M_{i,k}} p_{i,k,t}^{j,m} y_{i,k,t}^{j,m} \right)}{\sum_{k=1}^{K} \sum_{m=1}^{M_{i,k}} p_{i,k,t}^m y_{i,k,t}^m}. \qquad (15)$$

Equation (12) tells us that international trade has two pathways by which it increases variation in GDP growth. The first pathway is that trade increases the Herfindahl index of the economy through specialization in more productive industries, where the productivity of industries is determined by the productivity of firms within an industry. The second pathway is that it amplifies the impact of shocks to firms in traded industries, which, similarly to the effect on the Herfindahl index, is because trade breaks the restriction that production must be constant within each traded industry. This implies that firms are able to steal the market shares of not only firms within the same industry and country, but also the market shares of firms in other countries; although due to general equilibrium effects and balanced trade, the country must lose market share in other traded industries. Despite the offsetting effect of losing market share in other industries, international trade still amplifies variation in constant price GDP growth, as

the firms that gain the most market share, and hence labor input, will be the firms that experience the highest productivity growth, while the firms that lose market share will be firms that experience relatively low productivity growth. This addition amplifying effect is given by $\psi(\epsilon, S_{i,0})$, the behavior of which is illustrated in figure 2.

IV. Discussion of Theoretical Results

The mappings derived in section III shed insight on the forces governing how idiosyncratic shocks lead to variation in realized GDP growth due to granularity, or finiteness in the set of firms, in an otherwise standard model of growth and international trade. In these mappings, granularity does not impact the expected growth of an economy, but rather allows realized GDP growth to deviate from expected GDP growth, which does not happen when there is a continuum of firms. The four forces governing the magnitude of variation in GDP growth, as measured by the variance are: i) the growth process for firms, ii) how concentrated output is across firms, iii) how substitutable output is across firms, and iv) how much a country exports as a share of GDP.

Variation in the growth process for firms is essential for allowing granularity to generate variation in growth at the aggregate level. In this paper, I assume a Gibrat's law type proportional growth process, as this type of growth process matches observed firm growth in the data, is consistent with the observed distribution of firm sizes, and is a standard and flexible way for modeling the growth of firms. That idiosyncratic productivity shocks are both proportional and persistent (here permanent) is the reason why the model predicts higher variation in the growth of firms and therefore higher variation in GDP growth over longer periods of time as the result of granularity. If instead, shocks were assumed to be completely temporary, the framework not be able to explain why there is higher variation in the growth of firms, nor higher variation in GDP growth over longer periods of time.

How concentrated output is among firms is the same as in the partial equilibrium framework of Gabaix (2011), and is given by the sales Herfindahl of the economy. This is a measure of how granular the economy is, or how big the big firms are. In the case of a continuum of firms, the Herfindahl index of an economy will be equal to zero, implying zero variation in aggregate growth or that realized GDP growth will always equal expected GDP growth. International trade increases the Herfindahl index of the economy by allowing countries to specialize in the industries they are relatively most productive in, which is determined solely by the productivity of firms within each industry. The elasticity of substitution across firms similarly increases the Herfindahl index of an economy, as more productive firms will be larger if output is more substitutable across firms.

Outside of their direct effect on the concentration of output within an economy, both the elasticity of substitution and international trade further increase variation in GDP growth by amplifying the aggregate impact of idiosyncratic shocks to firms. A higher elasticity of substitution implies that firms that receive higher productivity shocks will growth more in terms of their output and labor input. International trade allows countries to allocate more labor to the traded industries which experience the largest productivity growth market and move it away from industries in which firms experiences the lowest productivity growth. This amplifying effect of international trade similarly increases as the elasticity of substitution increases, as the industries with the highest relative productivity growth will acquire more labor.

V. Estimation and Quantitative Results

In this section, I re-evaluate the question "can the aggregate variation generated by idiosyncratic shocks to firms be large enough to matter in practice?" in the context of the framework developed in this paper. Relative to previous papers in the granularity

literature, such as Gabaix (2011) and Carvalho and Grassi (2015), I broaden the scope of this question to focus not only on short-run macroeconomic fluctuations, but also whether idiosyncratic shocks to firms can be important in generating variation in GDP growth over longer periods of time. In particular, I focus on variation in 1 year GDP growth rates and 10 year GDP growth rates.

The goal of this section is to provide an answer for how much variation in GDP growth we might expect in a world where the only source of aggregate variation is due differences in the realizations of idiosyncratic productivity shocks to firms, noting that aggregate variation only arises in this context due to granularity and the existence of large firms. To provide such an answer, I carry out an experiment where I estimate the parameters of the model to construct the standard deviation of forecasted 1-year ahead GDP growth and forecasted 10-year ahead GDP growth for a given base year. I construct the standard deviations of these forecasts separately for each country, holding fixed the parameters governing the growth process for firms and the elasticity of substitution, as the derivation of (12) relies on these values being the same in all countries, while allowing the Herfindahl index and exports as a share of GDP to vary across countries.

V.A Firm-Level Data and Parameter Estimation

Exports as a share of GDP can be taken straight from aggregate data, for which I use the United Nations' (UN) World Development Indicators (WDI) database. To estimate the other parameters of (12), however, requires firm-level data on sales and gross margins. The firm-level data comes from the OSIRIS Industrials database. The database is compiled by Bureau Van Dijk, which is a private specialist provider of information on public and private companies worldwide, with a focus on standardizing data to allow for international comparisons. The dataset contains standardized and reported information on globally listed, as well as major non-listed, companies spanning 190 countries and 20

years and excludes banking and financial firms. The OSIRIS database is a subset of Bureau Van Dijk's ORBIS database, which contains information on 150 million firms worldwide. The coverage of the database is not complete, but offers good coverage for large firms, as documented by Ribeiro, Menghinello, and De Backer (2010), and these are the firms that are most important for the purposes of this paper.

I select 2005 to be the base year, as it has the largest number of active firms in the OSIRIS database and avoids entangling the parameter estimates with the effects of the 2008-2009 recession. I compute the sales Herfindahl for each country following the definition from (13), where the denominator is from aggregate GDP data, again from the UN WDI database, as $\sum_{k=1}^{K} \sum_{m=1}^{M_{i,k}} p_{i,k,t}^m y_{i,k,t}^m = E_{i,0}$. In my framework, gross output is equal to GDP, however Gabaix (2011) and Hulten (1978) show that sales over GDP, and not sales over gross output or value added over GDP, is the correct measure of firm size for weighting the impact of microeconomic shocks on TFP or GDP when economies feature horizontal linkages across firms. To reduce the effects of a potential outlier in annual revenues for a firm, when computing the Herfindahl index for each country I use the median value of each firm's revenues between 2004–2006. Table 1 reports the square root of the estimated Herfindahl index for each country with at least 100 firms with positive revenues in 2005. There is significant cross-country variation in the number of firms with listed data in the OSIRIS database, however, Herfindahl indices computed using only the largest 100 firms for each country are nearly identical to the Herfindahl indices computed using all firms.

The next parameter that must be estimated is the elasticity of substitution across varieties, which I assume to be the same across countries and equal to the estimate for the United States. In my framework, the elasticity of substitution governs markups, and can be estimated using data on gross margins. In particular, we can estimate the elasticity as

$$\epsilon = \frac{1}{GM}, \qquad (16)$$

where GM is the gross margin of a firm:

$$GM := \frac{py - wl}{py}. \qquad (17)$$

In the data, gross margins vary significantly across firms. Therefore, I estimate the elasticity separately for each firm, and then take a revenue weighted average to arrive at an estimated elasticity of $\tilde{\epsilon} = 3.35$. This estimate is in line with what other papers have found and used, for example by Anderson and van Wincoop (2004), and is stable across different choices of base years.

The final parameters to estimate are the drift and volatility of the productivity growth process for firms, which are assumed to be the same in all countries and again equal to the estimates for the United States. As I do not have firm-level data on quantities and inputs, I am unable to observe productivity directly. Instead, I estimate the productivity process by combining the estimated elasticity of substitution with observed changes in firm level sales, aggregate expenditures, and consumer price indices. Logged growth in equilibrium sales for firm m as a fraction of industry expenditures is given by:

$$\log \Delta \left(\frac{p_{i,k,T}^m y_{i,k,T}^m}{\theta_k E_{i,T}} \right) = \log \Delta \left(\left(z_{i,k,T}^m \right)^{\epsilon-1} \right) + \log \Delta \left(\sum_{j=1}^{N} \sum_{m=1}^{M_{i,k}} \left(z_{i,k,T}^m \right)^{\epsilon-1} \right), \qquad (18)$$

where

$$\Delta \left(z_{i,k,T}^m \right)^{\epsilon-1} := \frac{\left(z_{i,k,T}^m \right)^{\epsilon-1}}{\left(z_{i,k,0}^m \right)^{\epsilon-1}}$$

and similarly for the other variables. Growth in the equilibrium consumption price index is given by

$$\log \Delta P^C_{i,k,T} = -\left(\frac{1}{\epsilon-1}\right) \log \Delta \left(\sum_{j=1}^{N}\sum_{m=1}^{M_{i,k}} \left(z^m_{i,k,T}\right)^{\epsilon-1}\right), \quad (19)$$

which substituting into equation (18), yields the equation by which we estimate the productivity process for firms:

$$\log \Delta z^m_{i,k,T} = \left(\frac{1}{\epsilon-1}\right)\left(\log \Delta\left(p^m_{i,k,T} y^m_{i,k,T}\right) - \log \Delta E_{i,T}\right) - \log \Delta P^C_{i,k,T}. \quad (20)$$

I compute logged annual productivity growth ($T = 1$) for each firm using equation (20) between 1995 and 2005. As Davis, Haltiwanger, Jarmin, and Miranda (2007) show, small firms are significantly more volatile than large firms, therefore I restrict my sample to large firms, where I define "large" to be firms with at least one year of over \$1 billion USD in total revenue during the sample period. Estimates of the drift, $\tilde{\mu}_{GBM}$, and volatility, $\tilde{\sigma}_{GBM}$, of the geometric Brownian motion productivity process in equation (4) can be computed using the sample mean, $\tilde{\mu}_{\log z}$, and sample standard deviation, $\tilde{\sigma}_{\log z}$, of logged annual productivity growth according to:

$$\tilde{\sigma}_{GBM} = \tilde{\sigma}_{\log z}. \quad (21)$$

$$\tilde{\mu}_{GBM} = \tilde{\mu}_{\log z} + \frac{\tilde{\sigma}^2_{\log z}}{2}. \quad (22)$$

To avoid exaggerating the volatility of firm growth due to outliers, I drop the productivity shocks in the highest and lowest decile for each year when computing these estimates. I am then able to plug these estimates into equations (5) and (6) to yield estimates for $E[X_T]$ and $Var[X_T]$.

The estimates for $T = 1$ and $T = 10$ are reported in table 2, in particular I find a standard deviation for 1 year firm-level productivity growth of 9.00 percent and of 33.99 percent for 10 year productivity growth. The 1 year estimates are similar in magnitude to alternative measures of volatility for large firms found elsewhere in the literature, for example in Gabaix (2011) or Davis, Haltiwanger, Jarmin, and Miranda (2007). If I instead

use $T = 10$ in (20) to directly compute 10-year productivity growth rates, I find a standard deviation for 10-year firm productivity growth of 33.70 percent, which is nearly identical to the original estimate obtained by substituting $T = 10$ into (6) using the drift and volatility from annual firm productivity growth rates. This equivalency highlights the consistency of geometric Brownian motion in matching observed firm growth rates.

V.B Quantitative Results and Discussion

To quantitatively evaluate the variation in GDP growth that arises due to granularity and idiosyncratic shocks to firms in my framework, I plug the estimated parameters from the previous section into (12) for each country in my sample. I focus on the standard deviation of the of 1-year GDP growth rate forecast to reveal how granularity can generate short-run macroeconomic fluctuations, and the standard deviation of the 10-year GDP growth rate forecast to focus on how granularity can lead to variation in longer-run growth trends.

The way to interpret this exercise, is that I use (12) to estimate the standard deviation for the distribution of potential forecasted GDP growth for 2005–2006 (1-year) and 2005–2015 (10-year) for each country, where I assume the only source of aggregate variation is due to granularity and variation in the realizations of idiosyncratic productivity shocks to firms. The results of these exercises are in tables 3 and 4. Note that in this paper, I do not exploit the fact that variation in GDP growth can arise due to variation in the growth of foreign countries, as I effectively assume that the world as a whole is non-granular. Therefore, for each country, all relevant information for the state of the rest of the world is contained in the Herfindahl index of a country and exports as a share of GDP.

To put my quantitative results in context, I compare the magnitude of the standard deviations for forecasted GDP growth to the magnitude of observed historic volatility in

GDP growth for each country. I use GDP per capita, instead of GDP, to partially adjust for aggregate variation that arises due to population dynamics, which is not an object of focus in my framework. I compute the volatility of 1-year GDP growth rates over the period 1990-2005 and the volatility of 10-year GDP growth rates over the period 1950-2010 using data on constant price (PPP) GDP per capita from the Penn World Tables. Note that in the data, the base parameters are not fixed over time. In particular, one should expect small changes in the Herfindahl index across periods, meaning this is not a direct comparison to the fixed base year experiment of the model. Nevertheless, this serves as a useful metric for gauging whether the aggregate variation generated by idiosyncratic shocks in the model is large enough to demand attention in practice.

Let $\hat{\sigma}_{i,GDP,T}$ be observed volatility T-year GDP growth rates for each country. I compute

$$\gamma_{i,T} = \sqrt{\frac{Var[\Delta_{i,GDP,T}]}{\hat{\sigma}_{i,GDP,T}^2}}, \qquad (23)$$

to represent the relative magnitude of variation in GDP growth generated by idiosyncratic shocks in the model compared to the magnitude of variation observed in the data. Table 5 reports the mean and median of these ratios, broken down by whether countries are OECD (Organisation for Economic Co-operation and Development) members or not. For OECD countries, for $T = 1$, I find the average value of $\gamma_{i,1}$ to be 69.1 percent, while for $T = 10$ the average value of $\gamma_{i,10}$ is 44.8 percent. These ratios are lower for non-OECD members, which suggests that idiosyncratic shocks to firms have less potential to explaining the volatility of developing countries compared to developed countries. Table 6 shows the full parameterization and results for the United States and Korea, where these countries are chosen as examples due to their significant differences in Herfindahl indices and exports as a share of GDP.

Roughly speaking, my results indicate that, in a world where idiosyncratic shocks to firms are the only source of aggregate variation, on average we would still expect around half the variation in GDP growth rates that we observe in the data for OECD economies, and for both 1-year GDP growth rates and 10-year GDP growth rates. This indicates that idiosyncratic shocks to firms can potentially play a significant role in explaining not only why countries grow more in some years than other, but also why countries can grow more in some decades than others, indicating a much broader role for the aggregate impact of idiosyncratic shocks to firms than previously realized by the granularity literature.

V.C Discussion on Causality

My quantitative results establish that idiosyncratic shocks to firms can potentially play a significant role in explaining observed variation in both short-run (1 year) and longer-run (10-year) GDP growth rates. This begs the question: to what extent do idiosyncratic shocks to firms actually explain observed variation in GDP growth rates? Gabaix (2011) and Stella (2014) take opposing viewpoints on this matter in the context of short-run macroeconomic fluctuations for the United States, with Gabaix arguing that idiosyncratic shocks to firms can explain a significant portion of observed 1-year U.S. GDP volatility and Stella arguing that they cannot. This paper sidesteps this debate, as the contribution of this paper is not establishing causality, but rather the scope and potential for causality. The question of casualty is best tackled independently for each country and time period, as the answer is likely context specific. For example, it is unlikely that the recent 2008-2009 recession can be significantly explained by idiosyncratic shocks to firms alone. Conversely, it is very plausible that idiosyncratic shocks to Nokia may explain a large portion of Finland's high growth through the early 2000's as well as Finland's subsequent recession, which has coincided with the rapid decline of Nokia.

Rather than attempting to establish causality with regards to whether idiosyncratic shocks to firms explain observed variation in GDP growth across different countries and time periods, the goal of this paper is to evaluate whether there is theoretical and quantitative merit to this hypothesis in the first place. In my framework, if the magnitude of GDP variation generated by idiosyncratic shocks to firms were insignificant relative to the magnitude of observed GDP variation, this would be evidence that economists are justified in ignoring the potentially causal role of idiosyncratic shocks to firms in explaining aggregate variation. Therefore one could largely bypass the question of causality all together, due to a strong prior that idiosyncratic shocks to firms cannot be responsible for observed aggregate variation, which until recently was the predominant viewpoint in macroeconomics. Instead, my results indicate that idiosyncratic shocks to firms should not be ignored at the aggregate level, and not only can they be an important source of short-run aggregate variation, as argued by Gabaix (2011), but also an important source of aggregate variation in the longer-run as well. This paper, therefore, provides strong theoretical evidence that ignoring the aggregate impact of idiosyncratic shocks to firms does not appear justified.

VI. Stylized Facts and Empirical Evidence

In this section, I provide evidence that the predictions of the model are consistent with several stylized facts of the data. In particular, I show that the countries the framework predicts should be more volatile (due to higher standard deviation of forecasted GDP growth) are in fact more volatile. I focus on OECD countries and 1-year GDP growth volatility. I regress the standard deviation of forecasted 1-year GDP growth (model) on observed historic volatility of GDP growth over 1990–2005 (data). The results of this regression are reported in table 7. The coefficient is positive and significant, with an R squared of 0.47, and the correlation between the model and the data is 0.69. Table 8 shows that when volatility in the data is computed over alternative time periods the

correlation between the model and data remains positive. The alternative regression with the intercept forced to zero is illustrated in figure 2, the benefit of this regression is that the slope has a natural interpretation. The slope of the no-intercept regression is 1.39, indicating that the standard deviation of GDP growth in the data is, on average, 39% higher than the standard deviation of GDP growth in the model.

Next, I focus on the model's prediction that countries which engage heavily in international trade should, ceteris paribus, exhibit more volatility in their GDP growth. In the model, trade monotonically increases GDP volatility both indirectly, though increasing the Herfindahl index of an economy, as well as directly, by amplifying the aggregate impact of idiosyncratic shocks to firms conditional on the Herfindahl index. The correlations in table 8 show that volatility in the data is positively correlated with both exports as a share of GDP in 2005 and the Herfindahl index of a country in 2005. Table 9 present the average volatility of annual GDP per capita growth over 1990-2005 when countries are grouped by their share of exports as a percentage of GDP. These results confirm the predictions of the model that, on average, GDP volatility should increase as countries export more a fraction of GDP. Table 10 shows that this holds when the sample of countries is restricted to OECD members, which will be the focus moving forward.

To distinguish the mechanisms by which international trade leads to increased aggregate variation from those of di Giovanni and Levchenko (2012), I show that this increase in volatility is not due solely to the effect of international trade on increasing a country's Herfindahl index. Table 8 shows that exports as a share of GDP is positively correlated with the Herfindahl index of a country. Table 11 groups OECD countries by their Herfindahl index in 2005 and shows that countries with Herfindahl indices experienced higher volatility of annual GDP growth over 1990–2005. To partially disentangle the effects of the Herfindahl index and exports/GDP, I first group countries

according to the magnitude of their Herfindahl index, and then within each grouping, I create a subgroup based on how much a country exports as a share of GDP. Table 12 presents the results of this exercise and shows that, while countries with higher Herfindahl indices experience higher volatility of GDP growth, countries with higher exports as a share of GDP still experience higher volatility when compared to countries with similar Herfindahl indices.

A key feature of the model is that it predicts higher variation in growth over longer periods of time. Table 3 previously established that this prediction is consistent with the data with regards to the relative magnitudes of 10-year GDP growth rate volatility versus 1-year GDP growth rate volatility. Figures 4 and 5 highlight this prediction of the model by using the parameterized model to create prediction intervals for forecasts of growth for the United States and Korea. The confidence intervals are wider for Korea, indicating the greater uncertainty involved in accurately forecasting growth in an economy that exports more and has more concentrated output among firms. Another way to interpret these forecasts and prediction intervals is that if we have a large number of countries similar to Korea, we should observe more variation in their growth if they are like Korea as opposed to if they are like the United States. To evaluate this prediction, I group OECD countries by the standard deviation of their forecasted 10-year GDP growth rates in the model. I compute GDP growth for each country over the ten year period of 1995–2005 and then take the standard deviation of GDP growth across countries within groups. Table 13 compares these results to the average 10-year forecast standard deviation delivered by the model. These results indicate that the set of countries that model predicted should exhibit more variation in 10-year GDP growth did in fact exhibit higher variation in their GDP growth over 1995–2005. Within each grouping, I compute the ratio of the average standard deviation in the model to the standard deviation across countries in the data and these results indicate that we would expect roughly half the variation in

10–year cross-country growth over for OECD countries. These results indicate that, similarly to how granularity and idiosyncratic shocks to firms can potentially explain a significant amount of observed variation across time for a given country as suggested by table 5, they can also potentially play a large role in explaining variation in GDP growth across countries for a given time period.

V. Conclusion

In this paper, I evaluated the extent to which idiosyncratic shocks to firms can generate variation in GDP growth in a general equilibrium environment featuring monopolistic competition and international trade. Theoretically, I generalized the theoretical results of Gabaix regarding aggregate volatility in granular economies to an environment in which shocks to firms affect the output of other firms in the economy through competitive forces. My theoretical results reveal novel pathways through which idiosyncratic shocks to firms can directly generate and amplify aggregate volatility. In particular, the degree to which aggregate fluctuations arise from idiosyncratic shocks to firms depends not only on the concentration of output among firms, as shown by Gabaix and di Giovanni and Levchenko (2012), but also on how substitutable output is among firms and the degree to which firms compete internationally versus domestically. My framework thus offers an explanation for why countries that engage heavily in international trade more are more volatile, and suggests a potential avenue for explaining why some sectors exhibit more volatility than others as noted by di Giovanni and Levchenko (2009) and Grassi and Imbs (2014).

Quantitatively, I find that, given observed firm sizes and dynamics, even if idiosyncratic shocks to firms were the only potential source of variation in GDP growth, we would still expect roughly half the magnitude of variation in GDP growth observed in the data across a wide range of countries. Notably, this holds not only for variation in annual GDP growth rates, but also for variation in decade-to-decade GDP growth rates. This indicates that idiosyncratic shocks can be an important source of not only short-run

macroeconomic fluctuations, as noted by Gabaix (2011) and Carvalho and Grassi (2015), but also variation in longer-run growth trends. This suggests that when economists are interested in why countries grow more in some periods than others, or why countries may react differently to similar policy interventions, a first order thing that needs to be considered is differences in the growth of a country's largest and fastest growing firms. To what extent, and how, the growth of these firms is shaped by political, social, economic, and environmental factors not addressed in this paper remains an exciting area for future research.

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Table 1
Herfindahl Indices of Countries in OSIRIS Database

Country	Code	Firm Count	Herf. Index	Largest Firm Share	Country	Code	Firm Count	Herf. Index	Largest Firm Share
Argentina	ARG	253	0.063	0.03	Jordan	JOR	171	0.225	0.17
Australia	AUS	1665	0.104	0.04	Japan	JPN	7042	0.115	0.04
Belgium	BEL	182	0.132	0.06	Korea	KOR	2316	0.207	0.09
Brazil	BRA	837	0.141	0.09	Kuwait	KWT	155	0.046	0.03
Canada	CAN	4334	0.103	0.02	Sri Lanka	LKA	163	0.032	0.01
Switzerland	CHE	260	0.301	0.18	Mongolia	MNG	157	0.035	0.03
Chile	CHL	711	0.239	0.09	Mexico	MEX	279	0.164	0.11
China	CHN	1920	0.080	0.04	Malaysia	MYS	1073	0.268	0.25
Colombia	COL	206	0.087	0.06	Nigeria	NGA	114	0.021	0.01
Cyprus	CYP	113	0.109	0.06	Netherlands	NLD	245	0.226	0.09
Germany	DEU	921	0.143	0.07	Norway	NOR	225	0.304	0.19
Denmark	DNK	167	0.151	0.13	N. Zealand	NZL	156	0.099	0.07
Egypt	EGY	736	0.058	0.04	Oman	OMN	113	0.037	0.02
Spain	ESP	198	0.103	0.05	Peru	PER	239	0.062	0.03
Finland	FIN	143	0.339	0.20	Philippines	PHL	213	0.078	0.04
France	FRA	953	0.153	0.08	Pakistan	PAK	329	0.057	0.04
U.K.	GBR	2456	0.275	0.16	Russia	RUS	371	0.147	0.07
Greece	GRC	288	0.067	0.03	Sweden	SWE	486	0.189	0.08
Hong Kong	HKG	229	0.327	0.17	Singapore	SGP	675	0.406	0.34
Indonesia	IDN	303	0.030	0.02	Thailand	THA	530	0.154	0.13
Ireland	IRL	113	0.216	0.08	Turkey	TUR	236	0.062	0.04
Israel	ISR	756	0.112	0.04	U.S.A.	USA	7742	0.057	0.03
India	IND	3284	0.055	0.04	Vietnam	VNM	341	0.022	0.02
Iran	IRN	179	0.028	0.02	S. Africa	ZAF	363	0.141	0.08
Italy	ITA	297	0.095	0.05					

The column *Herf Index* reports the square root of the estimated Herfindahl index for each country in the OSIRIS database with over 100 firms with positive revenues in 2005. The column *Largest Firm Share* reports the 2005 revenues of the largest firm in each country divided by the 2005 GDP of that country. *Firm Count* lists the number of firms with positive 2005 revenues for each country. The average Herfindahl index among the countries in table 1 is 0.136, while the revenues of the largest firm in each country are on average 7.9 percent of GDP.

In total, the OSIRIS database contains 51,287 firms headquartered in 136 countries with positive revenues in 2005. The total revenues of these firms sum up to 95.2 percent of 2005 World GDP and yield a global Herfindahl index of 0.028.

Table 2

Estimated Productivity Growth Process for Firms

Estimated Object	Variable	Estimated Value
Mean Logged Annual Productivity Growth	$\tilde{\mu}_{\log z}$	0.0105
St.Dev. of Logged Annual Productivity Growth	$\tilde{\sigma}_{\log z}$	0.0885
Drift of GBM Process	$\tilde{\mu}_{GBM}$	0.0145
Volatility of GBM Process	$\tilde{\sigma}_{GBM}$	0.0885
Expected 1-year Firm Growth	$E[X_1] - 1$	1.46%
St.Dev. of Forecasted 1-Year Firm Growth	$\sqrt{Var[X_1]}$	9.00%
Expected 10-year Firm Growth	$E[X_{10}] - 1$	15.55%
St.Dev. of Forecasted 10-Year Firm Growth	$\sqrt{Var[X_{10}]}$	32.99%

Table 2 reports the estimated parameters governing the productivity growth process for firms. Logged annual productivity growth for each firm between 1995–2005 is computed using equation (20) using an estimated elasticity of substitution of 3.35. When computing the sample mean and sample standard deviation, the sample is restricted to U.S. firms with at least 1 year of greater than $1 Billion USD in sales over the sample period. These estimates are used along with equations (21) and (22) to estimate the drift and volatility of the geometric Brownian motion process in equation (4). The drift and volatility is then used to estimate the expectation and standard deviation of productivity growth for each firm after both 1-year and 10-years using equations (5) and (6).

If I instead used logged 10-year productivity growth rates directly to estimate the standard deviation of 10-year firm growth I arrive at an estimate of 33.70 percent, which is nearly identical to the estimated 10-year standard deviation above of 32.99 percent, which uses annual growth rates.

Table 3
Standard Deviation of Forecasted 1-Year GDP Growth
Versus Historic 1-Year GDP Growth Volatility (1990-2005)

Country	Herf. Index	Exports /GDP	Pred St.Dev.	Data Vol.	Country	Herf. Index	Exports /GDP	Pred St.Dev.	Data Vol.
Argentina	0.063	20.95	0.71	10.97	Jordan	0.225	52.95	2.99	9.86
Australia	0.103	18.22	1.13	1.65	Japan	0.115	14.56	1.22	2.30
Belgium	0.132	73.37	1.87	3.64	Korea	0.207	37.42	2.56	5.02
Brazil	0.141	15.31	1.51	7.18	Kuwait	0.046	62.15	0.63	53.65
Canada	0.103	36.63	1.26	2.54	Sri Lanka	0.032	32.6	0.39	3.79
Switzerland	0.301	54.07	4.01	1.83	Mexico	0.164	26.79	1.90	4.50
Chile	0.239	39.56	2.99	5.58	Mongolia	0.035	59.67	0.47	13.52
China	0.080	36.77	0.99	3.65	Malaysia	0.268	113.49	3.98	7.24
Colombia	0.087	17.09	0.95	3.39	Nigeria	0.021	34.98	0.26	21.26
Cyprus	0.109	48.04	1.42	6.52	Netherlands	0.226	66.35	3.14	3.42
Germany	0.143	38.17	1.78	1.58	Norway	0.304	43.75	3.88	3.96
Denmark	0.151	47.24	1.96	2.51	N. Zealand	0.099	29.43	1.18	2.40
Egypt	0.058	29.51	0.69	7.55	Oman	0.037	55.28	0.49	7.47
Spain	0.103	24.91	1.19	2.37	Pakistan	0.057	15.16	0.61	3.26
Finland	0.339	40.67	4.27	5.02	Peru	0.062	26.32	0.72	4.95
France	0.153	26.48	1.78	2.76	Philippines	0.078	47.1	1.01	6.15
U.K.	0.275	25.98	3.18	2.62	Russia	0.147	34.45	1.79	12.14
Greece	0.067	21.10	0.75	2.6	Singapore	0.406	224.17	6.03	12.66
Hong Kong	0.327	194.37	4.85	7.48	Sweden	0.189	45.84	2.44	3.56
Indonesia	0.030	32.44	0.37	7.36	Thailand	0.154	72.64	2.19	7.91
India	0.055	19.30	0.61	2.63	Turkey	0.062	22.69	0.70	7.16
Ireland	0.216	78.45	3.11	3.62	U.S.A.	0.057	10.09	0.58	1.43
Iran	0.028	31.52	0.33	5.69	Vietnam	0.022	62.11	0.30	3.66
Israel	0.112	40.48	1.41	3.66	S. Africa	0.141	27.94	1.65	2.67
Italy	0.095	24.97	1.10	2.31					

Column *Pred St.Dev.* reports the standard deviation (percent) of forecasted 1-year GDP growth in my framework where idiosyncratic shocks to firms are the only source of variation/ uncertainty in GDP growth. These values are computed using equation (12) using $T = 1$, where the parameters governing the growth process for firms are listed in table 2 and the estimated elasticity of substitution is 3.35. *Herf. Index* lists the Herfindahl indices from table 1 for each country. *Exports/GDP* is the share of exports as a percentage of GDP in 2005, and comes from the UN's WDI database. *Data Vol.* is the observed volatility (st.dev.) of annual GDP growth rates between 1990–2005 (percent). These are computed using annual constant price (PPP) GDP per Capita data from the Penn World Tables (PWT 8.0).

Table 4
Standard Deviation of Forecasted 10-Year GDP Growth
Versus Historic 10-Year GDP Growth Volatility (1950-2010)

Country	Herf. Index	Exports /GDP	Pred St.Dev.	Data Vol.	Country	Herf. Index	Exports /GDP	Pred St.Dev.	Data Vol.
Argentina	0.063	20.95	2.71	38.59	Jordan	0.225	52.95	11.48	36.51
Australia	0.103	18.22	4.34	12.22	Japan	0.115	14.56	4.67	69.36
Belgium	0.132	73.37	7.19	15.27	Korea	0.207	37.42	9.82	40.24
Brazil	0.141	15.31	5.79	22.33	Kuwait	0.046	62.15	2.41	98.93
Canada	0.103	36.63	4.85	14.30	Sri Lanka	0.032	32.60	1.48	24.88
Switzerland	0.301	54.07	15.40	13.53	Mexico	0.164	26.79	7.31	20.65
Chile	0.239	39.56	11.49	17.78	Mongolia	0.035	59.67	1.80	63.92
China	0.08	36.77	3.80	44.10	Malaysia	0.268	113.49	15.28	19.30
Colombia	0.087	17.09	3.63	22.95	Nigeria	0.021	34.98	0.99	101.61
Cyprus	0.109	48.04	5.44	31.89	Netherlands	0.226	66.35	12.07	15.68
Germany	0.143	38.17	6.83	34.69	Norway	0.304	43.75	14.91	15.43
Denmark	0.151	47.24	7.53	15.00	N. Zealand	0.099	29.43	4.51	11.75
Egypt	0.058	29.51	2.65	43.19	Oman	0.037	55.28	1.89	42.80
Spain	0.103	24.91	4.56	27.37	Pakistan	0.057	15.16	2.33	19.38
Finland	0.339	40.67	16.39	17.59	Peru	0.062	26.32	2.76	30.53
France	0.153	26.48	6.82	20.64	Philippines	0.078	47.10	3.86	17.61
U.K.	0.275	25.98	12.23	11.95	Russia	0.147	34.45	6.87	--
Greece	0.067	21.10	2.87	38.64	Singapore	0.406	224.17	23.15	41.62
Hong Kong	0.327	194.37	18.62	30.74	Sweden	0.189	45.84	9.35	14.16
Indonesia	0.03	32.44	1.41	22.77	Thailand	0.154	72.64	8.40	35.85
India	0.055	19.30	2.32	35.62	Turkey	0.062	22.69	2.68	13.53
Ireland	0.216	78.45	11.93	25.49	U.S.A.	0.057	10.09	2.23	10.06
Iran	0.028	31.52	1.28	81.72	Vietnam	0.022	62.11	1.15	32.89
Israel	0.112	40.48	5.41	53.20	S. Africa	0.141	27.94	6.33	17.70
Italy	0.095	24.97	4.21	29.91					

Column *Pred St.Dev.* reports the standard deviation (percent) of forecasted 10-year GDP growth in my framework where idiosyncratic shocks to firms are the only source of variation/ uncertainty in GDP growth. These values are computed using equation (12) using $T = 10$, where the parameters governing the growth process for firms are listed in table 2 and the estimated elasticity of substitution is 3.35. *Herf. Index* lists the Herfindahl indices from table 1 for each country. *Exports/GDP* is the share of exports as a percentage of GDP in 2005, and comes from the UN's WDI database. *Data Vol.* is the observed volatility (percent) of 10-year GDP growth rates between 1950–2010 (GDP growth between 1950–1960, 1960–1970,…). These are computed using annual constant price (PPP) GDP per Capita data from the Penn World Tables (PWT 8.0). Observed volatility is not computed for Russia, as it has only two decades of data availability, while every other country listed has at least four decades of data availability.

Table 5
Summary for Ratios of Model/Historic GDP Volatility
1-Year vs 10-Year and OECD vs non-OECD

Sample	Object	Variable	Mean Value	Median Value
All	1-Year Ratio	$\gamma_{i,1}$	47.4%	42.3%
(45 Countries)	10-Year Ratio	$\gamma_{i,10}$	33.6%	23.4%
OECD	1-Year Ratio	$\gamma_{i,1}$	69.1%	53.3%
(24 Countries)	10-Year Ratio	$\gamma_{i,10}$	44.8%	35.5%
Non-OECD	1-Year Ratio	$\gamma_{i,1}$	22.6%	18.6%
(21 Countries)	10-Year Ratio	$\gamma_{i,10}$	20.9%	12.0%

Table 5 reports the mean and median values of the ratio of the standard deviation of forecasted GDP growth to observed historic volatility of GDP growth for both 1-Year GDP growth rates and 10-year GDP growth rates. These ratios are computed for each country according to (23). The standard deviation of forecasted GDP growth is computed using equation (12) with $T = 1$ for 1-year GDP growth rates and $T = 10$ for 10-year GDP growth rates. Observed historic 1-year GDP volatility is for 1990-2005, while 10-year GDP volatility is for 1950-2010, these values are reported in tables 3 and 4, respectively. When computing the mean and median of the ratios, I exclude the countries in the top decile of observed GDP volatility as outliers (all non-OECD). These outlier countries have an average volatility over 500 percent greater than the volatility of non-outlier countries for 1-year GDP growth rate volatility and over 250 percent greater for 10-year GDP growth rate volatility.

A full list of OECD member countries is available at:
http://www.oecd.org/about/membersandpartners/list-oecd-member-countries.htm

Table 6

Parameter Estimates and Quantitative Results for the United States and Korea

Parameters/Results	Variable	United States	Korea
Drift of GBM Process	$\tilde{\mu}_{GBM}$	0.015	0.015
Volatility of GBM Process	$\tilde{\sigma}_{GBM}$	0.089	0.089
Elasticity of Substitution	ϵ	3.35	3.35
Herfindahl index	$\sqrt{h_{i,0}}$	0.057	0.207
Exports/GDP	$S_{i,0}$	0.101	0.374
St.Dev. of 1-Year Forecast	$\sqrt{Var[\Delta_{i,GDP,1}]}$	0.58%	2.56%
1-Year Volatility (1990–2005)	--	1.43%	5.02%
Ratio of Forecasted/Historic	$\gamma_{i,1}$	0.405	0.510
St.Dev. of 10-Year Forecast	$\sqrt{Var[\Delta_{i,GDP,10}]}$	2.23%	9.38%
10-Year Volatility (1950–2010)	--	10.06%	40.24%
Ratio of Forecasted/Historic	$\gamma_{i,10}$	0.212	0.233

Table 6 reports the estimated parameters and standard deviation of forecasted 1-year and 10-year GDP growth using equation (12) for the cases of Korea and the United States. The Herfindahl index and Exports/GDP are from table 3, for a base year of 2005. The drift and volatility of the GBM process for firms are from table 2, and the elasticity of substitution is estimated using (16). Observed historic volatility for both 1-year GDP growth rates and 10-year GDP growth rates are from tables 3 and 4, respectively, and computed using data on GDP per capita (PPP) from the Penn World Tables. The ratios are computed according to (23) and give the standard deviation of forecasted GDP growth in the model (which is non-zero due to granularity) divided by observed historic GDP volatility.

Table 7

Predicted Versus Observed Annual GDP Volatility for OECD Countries, (1990–2005)

Coefficient	Estimate	Standard Error
Model	0.771	0.183
Constant	1.572	0.004
Observations		22
R-squared		0.47

Table 7 lists the estimated coefficients and standard errors of the regression: $data = \beta_0 + \beta_1 * model$. *Data* refers to observed annual GDP volatility (percent) between 1990–2005, while *model* refers to the standard deviation of forecasted 1-year GDP growth in the model, where both are from table 3.

Turkey and Switzerland are excluded from this regressions and following OECD-only tables as outliers. Turkey experienced low GDP growth between 1990–2000 followed by explosive GDP growth from 2000–2005, and this tale of two halves leads Turkey to have the highest GDP volatility of any OECD country over 1990–2005 by a significant amount. Switzerland is excluded to as its unusually low GDP volatility appears to be the result of being in a sustained great depression, see Kehoe and Ruhl (2005).

Table 8
Correlation with Observed Annual GDP Volatility
for OECD Countries, Various Time Periods

Time Period	Correlation with Model	Correlation with Exports	Correlation with Herf.
1990–2005	0.686	0.427	0.672
1980–2010	0.593	0.356	0.577
1950–2010	0.245	0.013	0.257
Correlation between Trade and Herf.			0.418
Observations			22

Correlation with Model lists the Pearson's correlation coefficients between the standard deviation of forecasted 1-year GDP growth (2005 base year) using equation (12), which are listed in table 3, and observed annual GDP growth volatility (per capita, constant price PPP) over alternative time periods for OECD countries. Switzerland and Turkey are excluded as outliers for this and subsequent tables as discussed in the footnote for table 7. *Correlation with Exports* gives the correlation coefficient between observed annual GDP growth volatility and Exports/GDP. *Correlation with Herf.* gives the correlation coefficient between observed annual GDP growth volatility and the Herfindahl index of a country. Both the Herfindahl index and Exports as a share of GDP for each country are listed in table 3. As predicted by the model, countries that engage heavily in trade tend to have larger Herfindahl indices.

Table 9
Volatility of Annual GDP Growth (1990-2005):
Countries Grouped by Share of Exports as % of GDP

Set of Countries (by Exports/GDP)	Exports Cutoff	Observations	Mean Volatility	Median Volatility
Top 25%	> 47%	12	5.43%	5.09%
Middle 50%	between	22	4.52%	3.87%
Bottom 25%	< 25%	11	4.09%	2.63%
Top 50%	> 32%	23	5.13%	3.96%
Bottom 50%	< 32%	22	4.16%	3.01%

See footnote under table 12 for description of tables 9-12.

Table 10
Volatility of Annual GDP Growth (1990-2005):
OECD Countries Grouped by Share of Exports as % of GDP

Set of Countries (by Exports/GDP)	Exports Cutoff	Observations	Mean Volatility	Median Volatility
Top 25%	> 41%	6	3.45%	3.59%
Middle 50%	between	11	3.45%*	2.76%
Bottom 25%	< 25%	5	2.07%	2.30%
Top 50%	> 36%	11	3.78%	3.64%
Bottom 50%	< 36%	11	2.50%	2.40%

*Mean volatility of the middle 50% is skewed upward by high observed volatilities for Israel and Chile. Both countries are near the cutoff with exports/GDP of approximately 40% in 2005.

Table 11
Volatility of Annual GDP Growth (1990-2005):
OECD Countries Grouped by Herfindahl Indices

Set of Countries (by Herf. Index)	Herf. Cutoff	Observations	Mean Volatility	Median Volatility
Top 25%	> 0.21	6	4.04%	3.79%
Middle 50%	between	11	3.05%	2.76%
Bottom 25%	< 0.10	5	2.26%	2.40%
Top 50%	> 0.14	11	3.87%	3.62%
Bottom 50%	< 0.14	11	2.41%	2.37%

Table 12
Volatility of Annual GDP Growth (1990-2005):
OECD Countries Grouped by Herfindahl and Share of Exports as % of GDP

Set of Countries (by Herf. Index)	Herf. Cutoff	Set of Countries (by Exports/GDP)	Exports Cutoff	Obs.	Mean Volatility	Median Volatility
Top 50%	> .014	Top 50%	> 36%	8	4.09%	3.79%
		Bottom 50%	< 36%	3	3.30%	2.75%
Bottom 50%	< .014	Top 50%	> 36%	3	2.96%	3.63%
		Bottom 50%	< 36%	8	2.20%	2.33%

Tables 9-12 present cross-tabulations reporting the means and medians across countries of observed volatility in annual GDP growth from 1990–2005 taken from table 3. I separate countries by quartiles based on both their share of exports as a percentage of GDP in 2005 and, for OECD economies, their Herfindahl index in 2005. I then compute the mean and median under different groupings of these quartiles. The results show that countries that export more and countries with higher Herfindahl indices tend to exhibit higher volatility.

Table 13

Variation in Cross-Country Growth (1995–2005):

OECD Countries Grouped by St.Dev. of Forecasted 10-Year GDP Growth

Set of Countries (by St.Dev. of Forecasted Growth)	Obs.	Average St.Dev. of Forecasted 10-year GDP Growth	St.Dev. of Growth Across Countries	Ratio
Top 25%	6	13.2%	28.3%	0.47
Middle 50%	11	6.8%	14.3%	0.48
Bottom 25%	5	3.6%	10.1%	0.36
Top 50%	11	10.9%	22.5%	0.48
Bottom 50%	11	4.7%	12.3%	0.38

Table 13 groups countries into quartiles by the standard deviation of their forecasted 10-year GDP growth rates from table 4 and arranges these quartiles into alternative groupings. *Average St.Dev. of Forecasted 10-Year GDP Growth* reports the average value of these standard deviations within each grouping.

For each country, I then compute observed constant price (PPP) GDP per capita growth over 1995-2005 from the Penn World Tables. *St.Dev. of Growth Across Countries* reports the standard deviation of these observed growth rates across countries within each grouping.

Ratio reports the ratio of the average standard deviation of forecasted 10-year GDP growth to the standard deviation of observed GDP growth over 1995–2005 across countries.

Figure 1
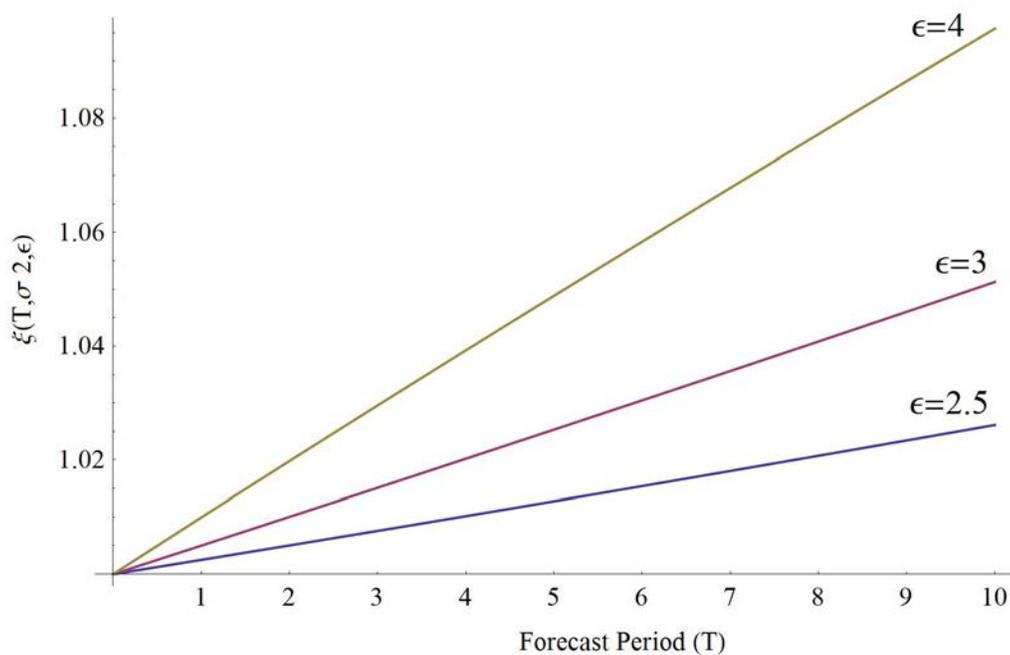**Behavior of Amplifying Curvature Term: $\xi(T, \sigma, \epsilon)$**

Figure 1 illustrates the behavior of $\xi(T, \sigma, \epsilon)$ from equation (10) for different values of ϵ and T. The volatility of the geometric Brownian motion process for firms is fixed at $\sigma = 0.10$. This figure indicates that the term is close to 1 and increases near linearly with T over short time periods, where the slope depends on the elasticity of substitution. If $\epsilon = 2$ then $\xi(T, \sigma, \epsilon) = 1$.

Figure 2
Behavior of Amplifying Trade Term: $\psi(\epsilon, S)$
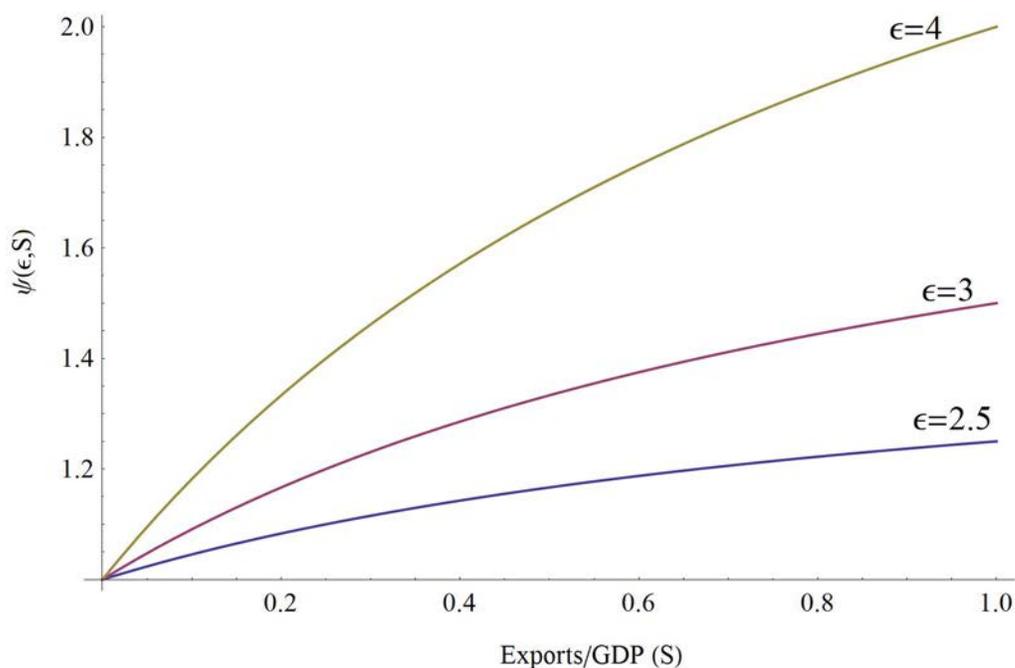
Figure 1 illustrates the behavior of $\psi(\epsilon, S)$ from equation (14) for different values of ϵ and S. International trade amplifies the variation in GDP growth by both increasing the Herfindahl index, which is not captured in this graph, and by amplifying the aggregate impact of idiosyncratic shocks to firms conditional on the Herfindahl index, which is given by this term This figure indicates that the term is monotonically increasing with a decreasing rate of increase as S approaches 1. The term increases as ϵ increases, indicating trade has a larger amplifying effect when output is more substitutable across firms. If $S = 0$ then there is no trade, indicating that $\psi(\epsilon, S) = 1$. The accuracy of this term depends on there being multiple traded sectors, as discussed in section III.E.i. If there is only a single traded sector then there is no amplifying effect of trade.

Figure 3

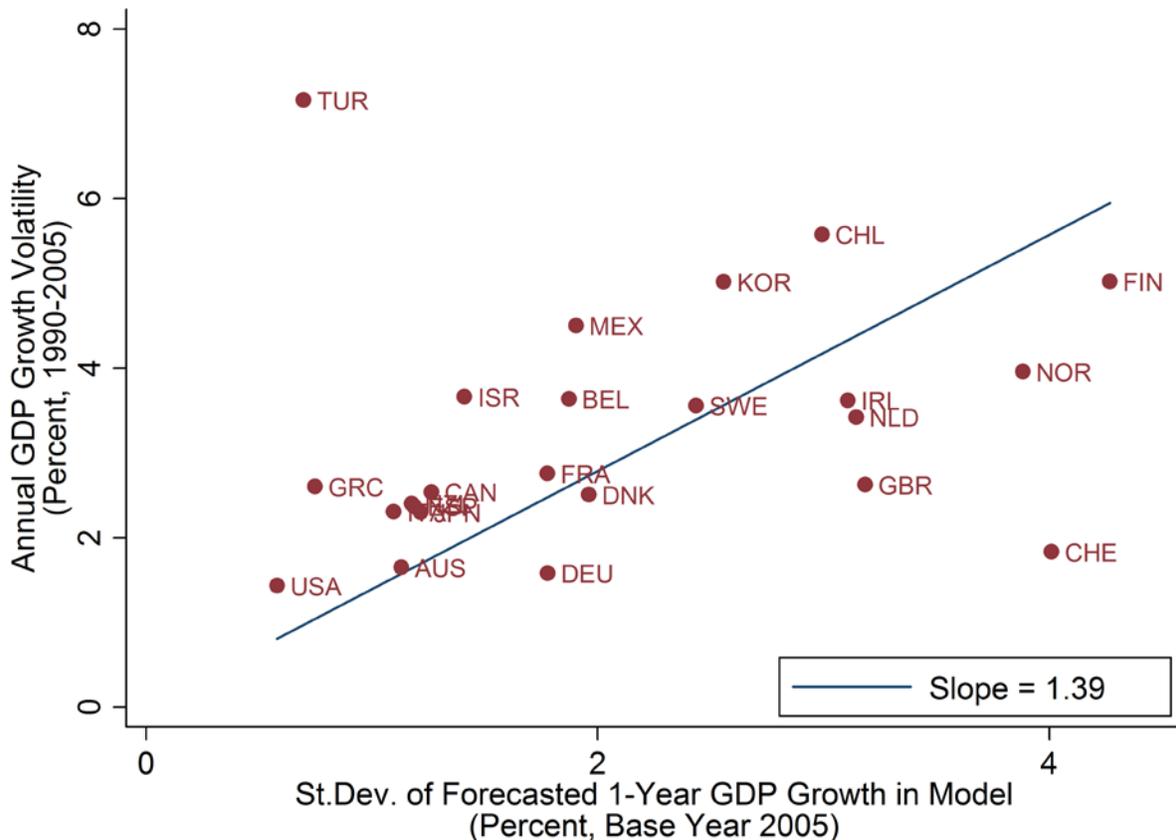**Predicted Versus Observed Annual GDP Volatility for OECD Countries, (1990–2005)**

Figure 3 plots observed annual GDP growth volatility (percent) between 1990–2005 versus the standard deviation of forecasted 1-year GDP growth in the model, where both are from table 3.

The fit line is from the following regression: $data = \beta_1 * model$. There is no constant in this regression. The interpretation of the slope is that the standard deviation of GDP growth in the data is, on average, 39% higher than the standard deviation of GDP growth in the model.

As detailed in table 7, Turkey and Switzerland are excluded from this regression as outliers. Turkey experienced low GDP growth between 1990–2000 followed by explosive GDP growth from 2000–2005, and this tale of two halves leads Turkey to have the highest GDP volatility of any OECD country over 1990–2005 by a significant amount. Switzerland is excluded as its unusually low GDP volatility appears to be the result of being in a sustained great depression, see Kehoe and Ruhl (2005).

Figure 4
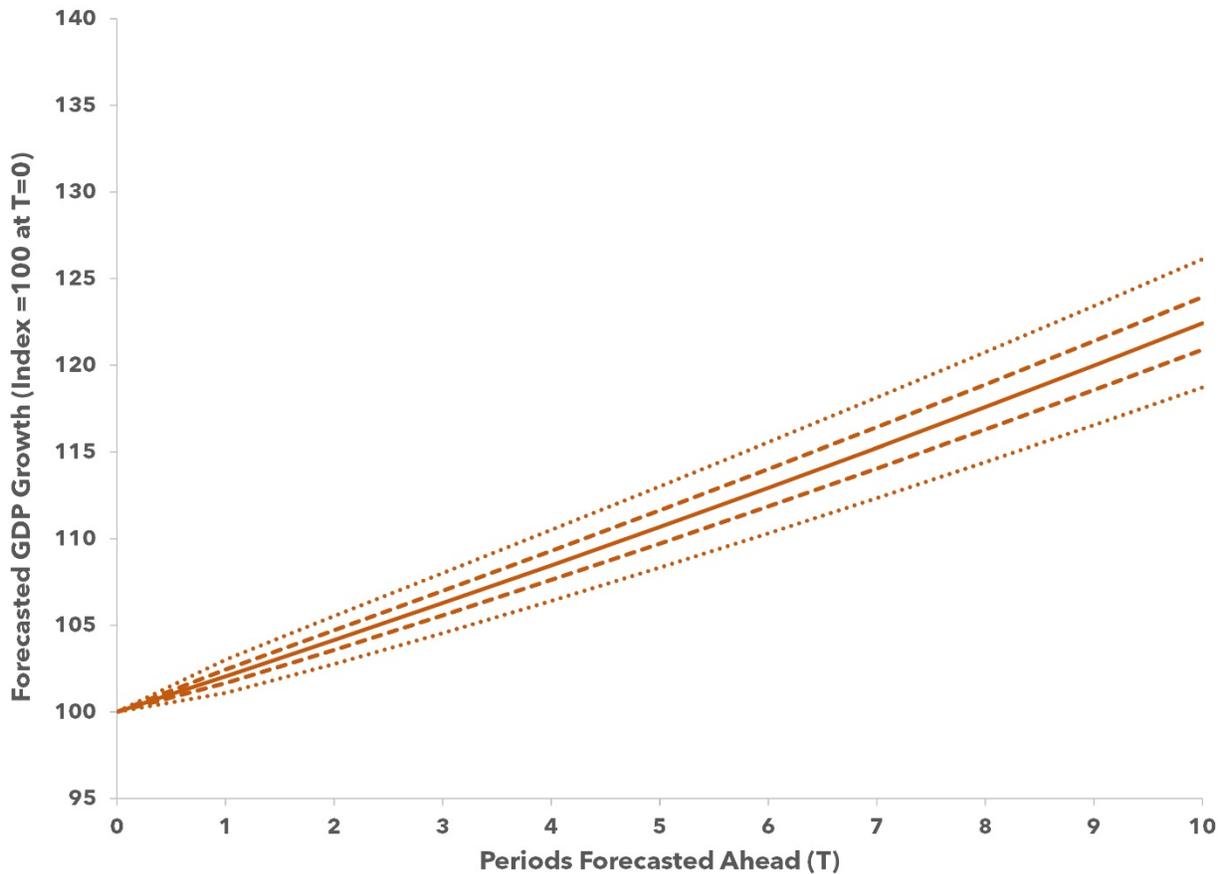**Forecasted GDP Growth for the United States**

Figure 4 presents forecasted growth using the model parameterized to the United States. The parameters used are taken from table 6 with a base year of 2005. Expected GDP growth is given by the solid line, and the dotted lines represent the prediction confidence intervals for the forecast. One interpretation of these intervals is as follows. Suppose we have a large number of countries exactly like the United States at $T = 0$ and sort them by realized GDP growth after 10 years. Due only to granularity and differences in the realizations of idiosyncratic shocks to individual firms, the country in the 95th percentile will have grown by 26% (2.3% annualized), the median country by 22% (2.0% annualized), and the country in the 5th percentile by 18% (1.7% annualized).
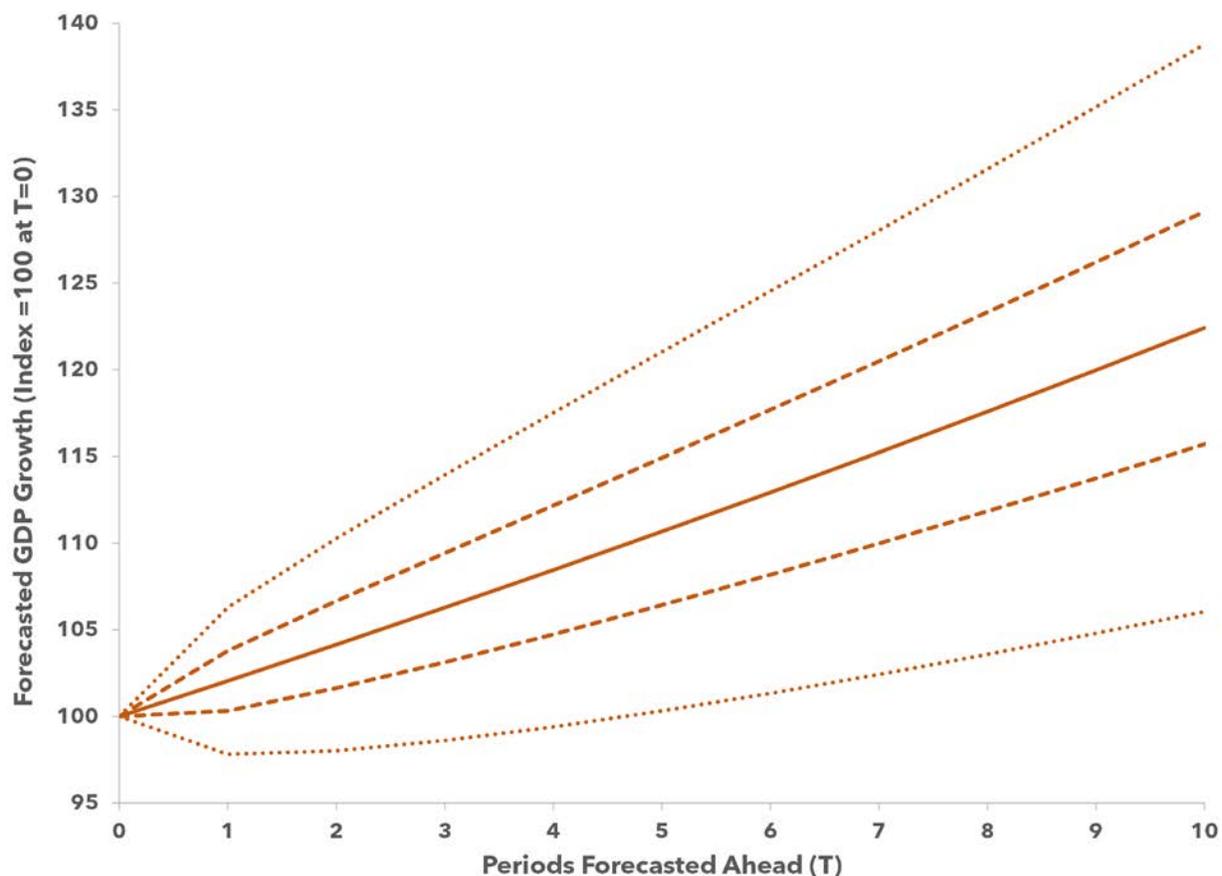
Figure 5
Forecasted GDP Growth for Korea

Figure 5 presents forecasted growth using the model parameterized to Korea. The parameters used are taken from table 6 with a base year of 2005. Expected GDP growth is given by the solid line, and the dotted lines represent the prediction confidence intervals for the forecast. One interpretation of these intervals is as follows. Suppose we have a large number of countries exactly like Korea at $T = 0$ and sort them by realized GDP growth after 10 years. Due only to granularity and differences in the realizations of idiosyncratic shocks to individual firms, the country in the 95th percentile will have grown by 38% (3.3% annualized), the median country by 22% (2.0% annualized), and the country in the 5th percentile by 5% (0.5% annualized).

Figure 6
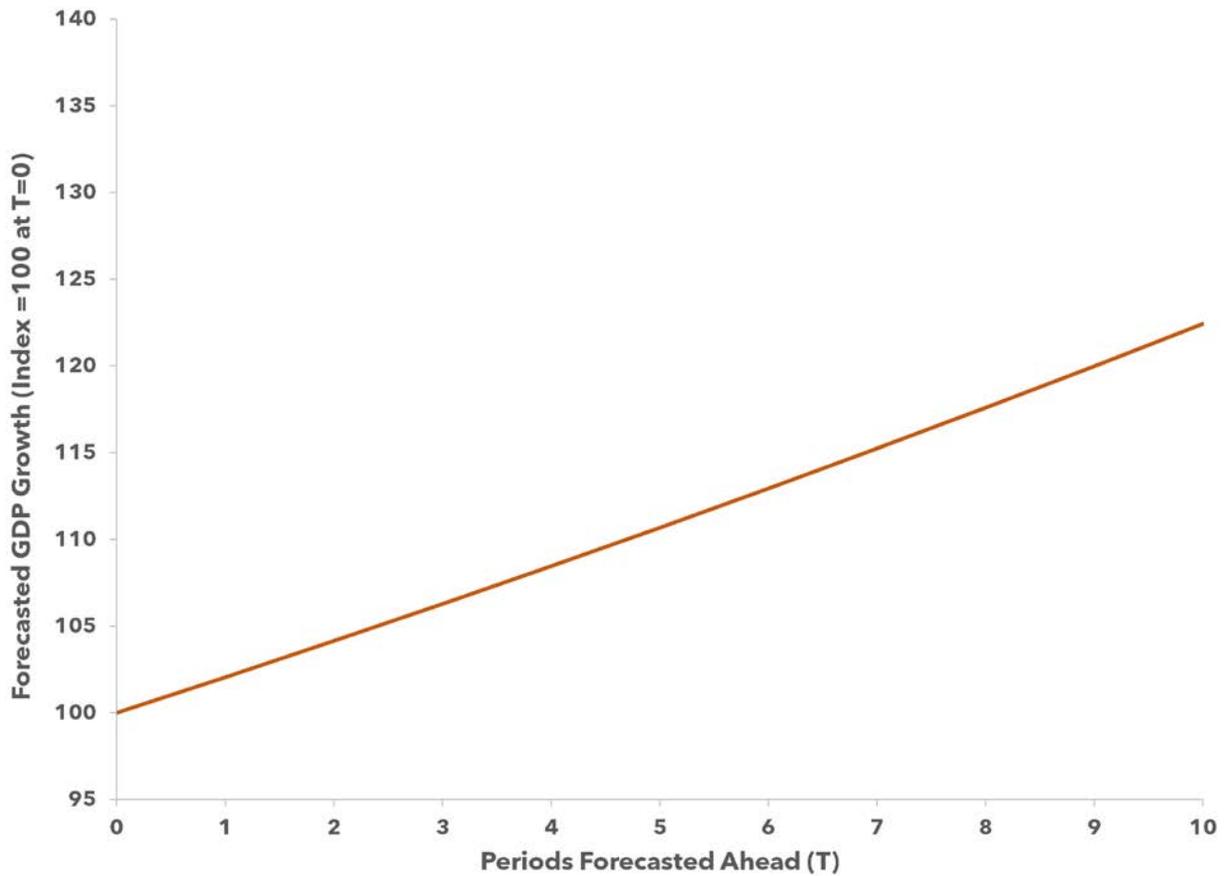**Forecasted GDP Growth for Non-Granular Economy**

Figure 6 presents forecasted growth in the model for a non-granular economy ($h_{i,0} = 0$) with the parameters for the growth process for firms and elasticity of substitution taken from table 6. In the absence of granularity, the model generates zero variation in GDP growth, which grows deterministically at 2.0 percent per year.