

ECON 256: Poverty, Growth & Inequality

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Population Growth and Economic Growth

Does population growth limit economic growth?

- Even now, we observe that countries with higher population growth rates tend to be poorer (although we've discussed that the causality isn't clear)

Malthusian Model

- Prior to 1800, there was relatively little growth in living standards and there were frequent famines that killed large numbers of people
- Thomas Malthus wrote an influential book in 1798 arguing that living standards would never increase, since population growth would offset any gains

Malthusian Model: Production Function

The Malthusian model assumes that a country has the following production technology

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- Y is the **output** a country can produce. Think of it as **real GDP** or real Output.

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 - If you double both L and N you double **output**. If you only double one input, you less than double **output**.
- z is the productivity or **TFP (Total Factor Productivity)** of the economy.
 - z represents technology and doubling z doubles **output**.

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Useful Trick:

- Doubling both F and N doubles output \Rightarrow use that insight and divide both L and N by N.

$$\frac{Y}{N} = z * F\left(\frac{L}{N}, \frac{N}{N}\right)$$

Or equivalently

$$\frac{Y}{N} = z * F\left(\frac{L}{N}, 1\right)$$

This is useful since it puts our production function in terms of GDP per capita or living standards

Malthusian Model: Population Growth

Assume that the population growth rate depends on the current standard of living per worker

$$\frac{\text{Population this Period}}{\text{Population this Period}} = g\left(\frac{\text{Output this Period}}{\text{Population this Period}}\right)$$

Which we write as (use an apostrophe, ' , to denote the value of a variable next period)

$$\frac{N'}{N} = g\left(\frac{Y}{N}\right)$$

- Where g is an increasing function (if you are richer you are able to have more children)

Malthusian Model: Population Growth

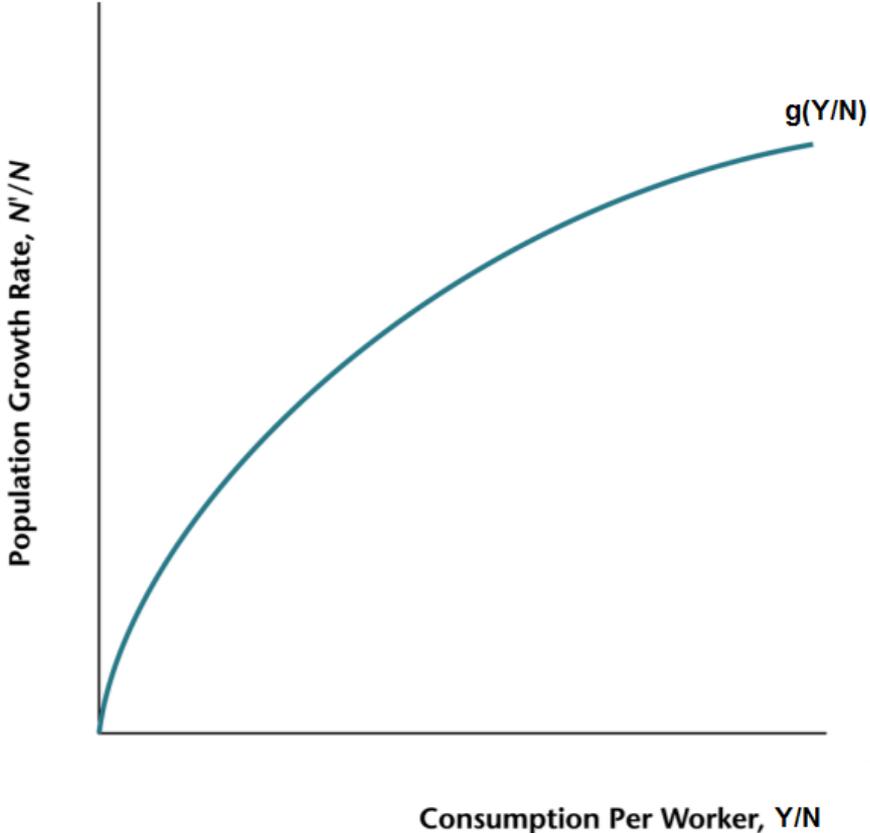


Figure from [Williamson](#)

Model Dynamics

The dynamics of the Malthusian model are pinned down by the following two equations

- Evolution of Living Standards

$$\frac{Y}{N} = z * F\left(\frac{L}{N}, 1\right)$$

- Evolution of Population Growth

$$\frac{N'}{N} = g\left(\frac{Y}{N}\right)$$

Given any initial values of z , L , and N (knowing the functions F and g), you can plug the values into the above two equations to see how the economy will evolve

Population and Steady State

Our population growth rate equation can be rewritten to give the level of population:

$$N' = g\left(\frac{Y}{N}\right)N$$

And we can also plug our expression for Y/N directly into the above

$$N' = g\left(z * F\left(\frac{L}{N}, 1\right)\right)N$$

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A country is in a **Steady State** when no variables change from year to year

- Steady State is when Population this year equals population next year: $N' = N$

Steady State Population

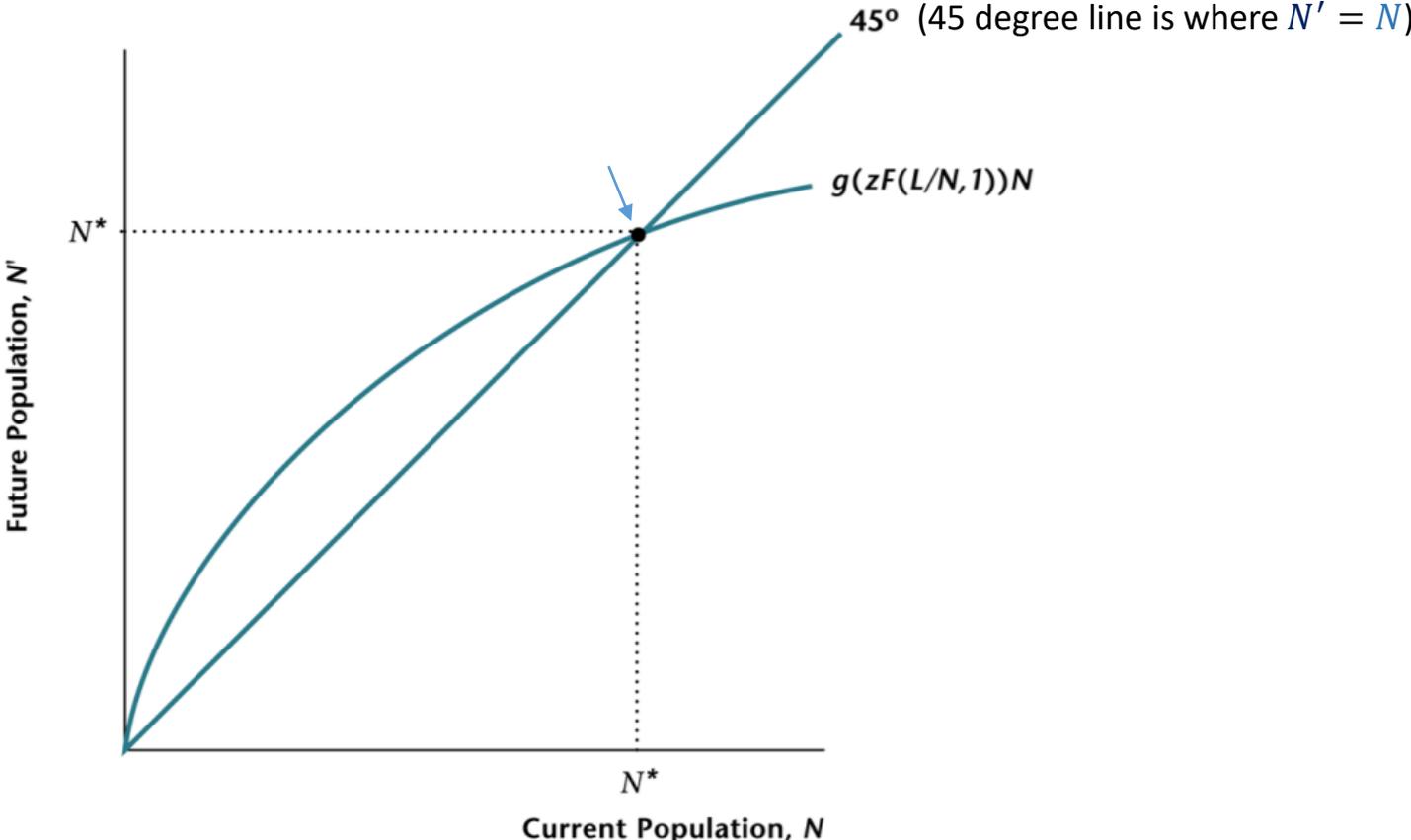
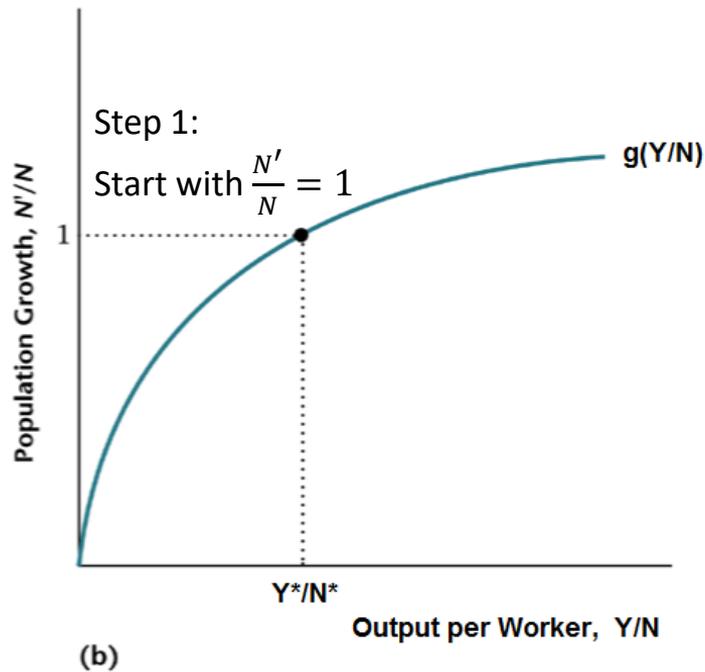
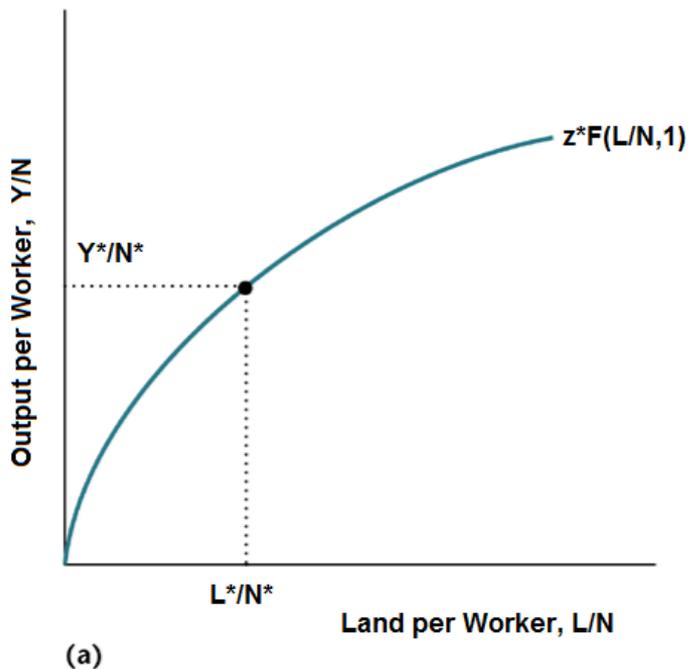


Figure from [Williamson](#)

Equilibrium

First we solve for the steady state value of population, N^*

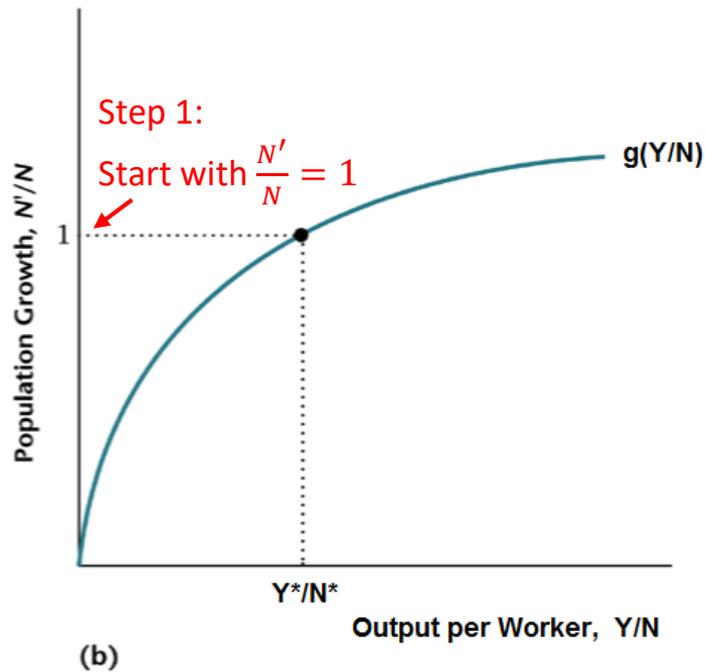
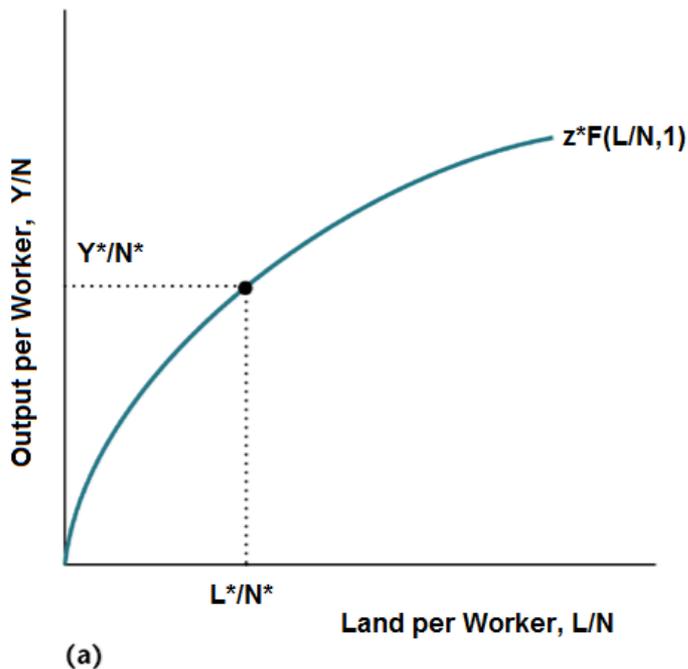
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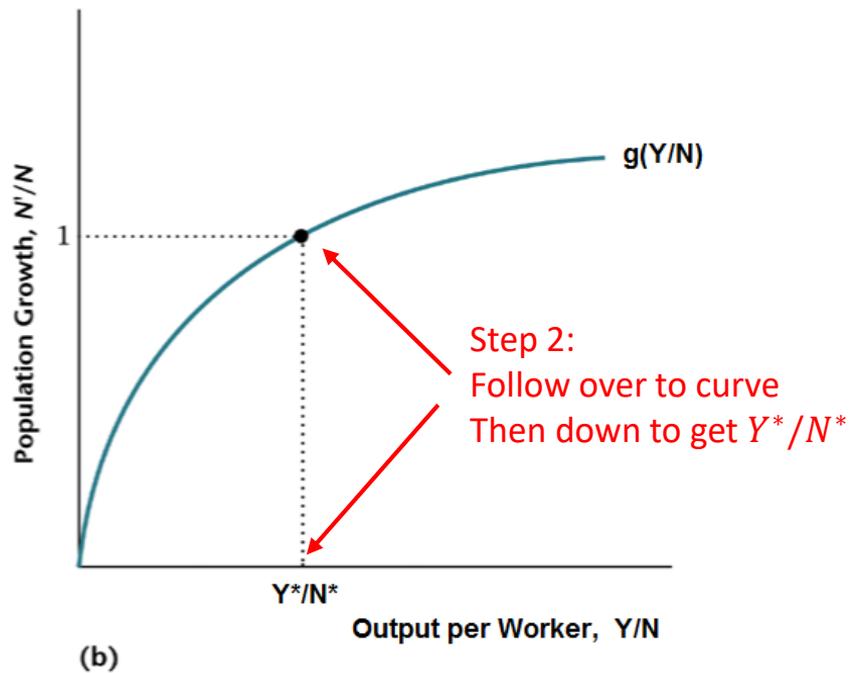
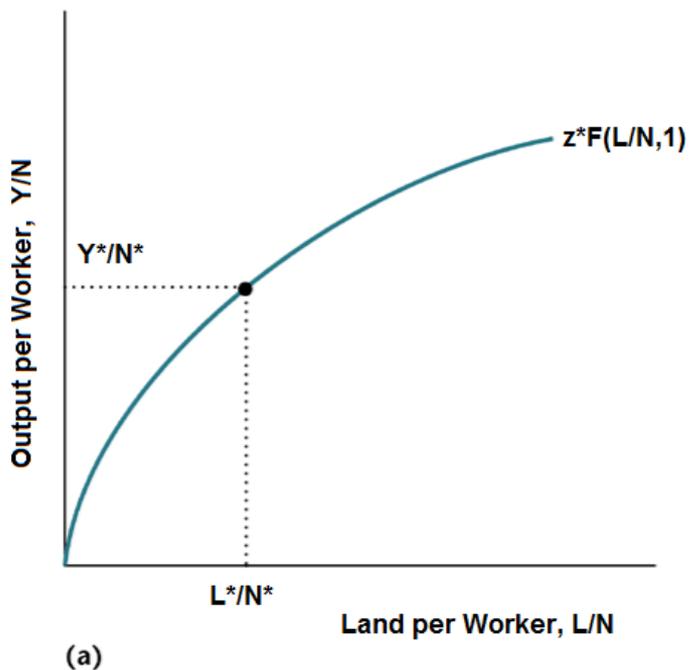
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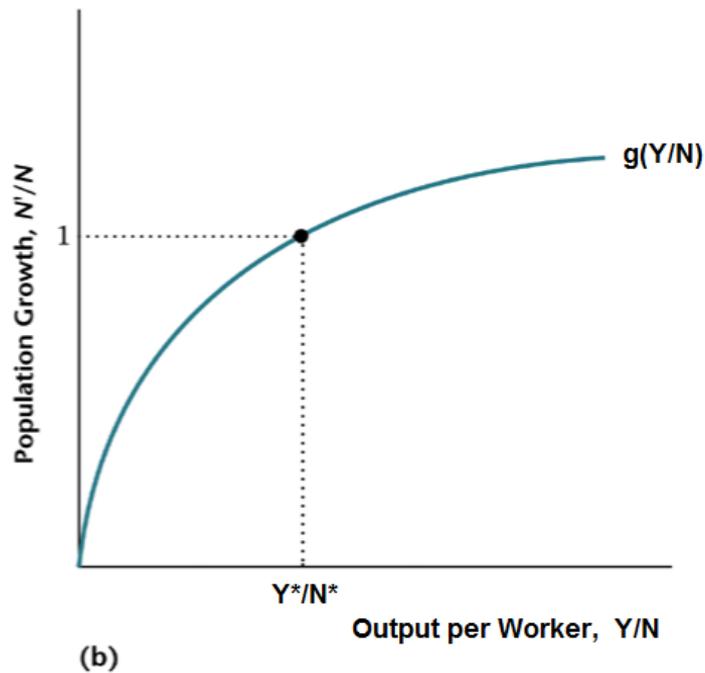
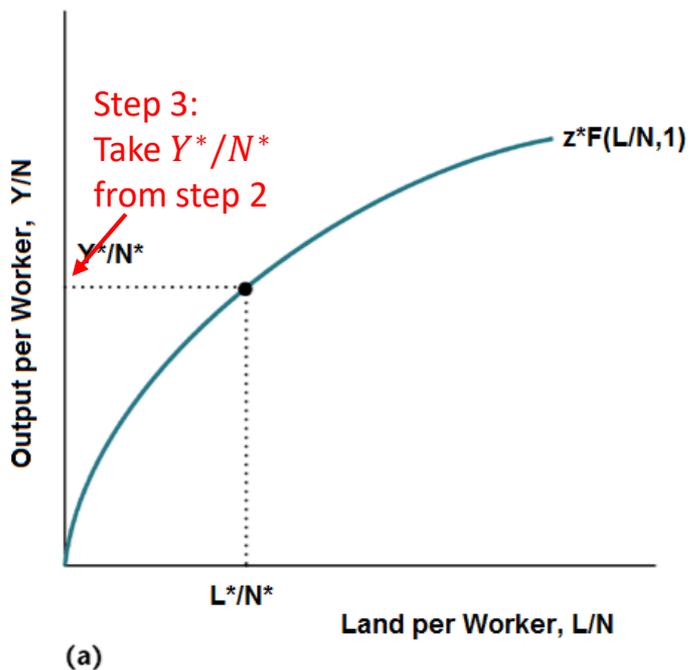
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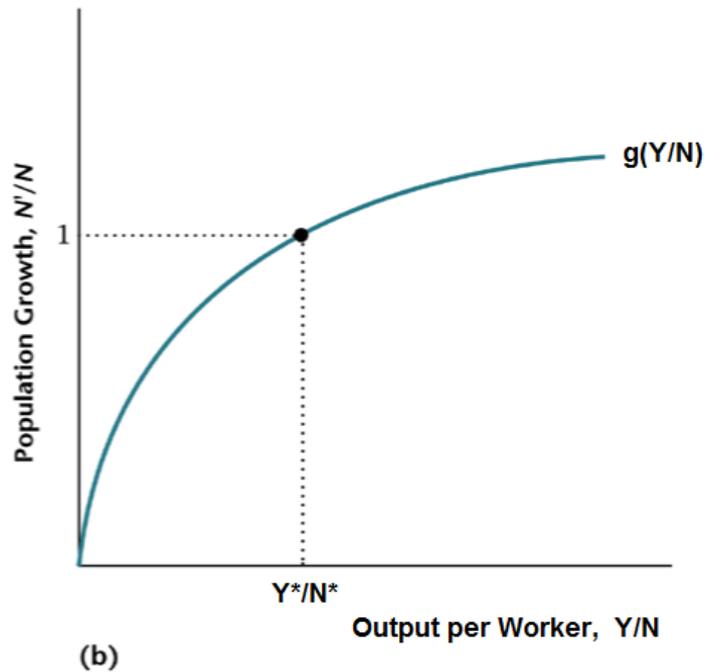
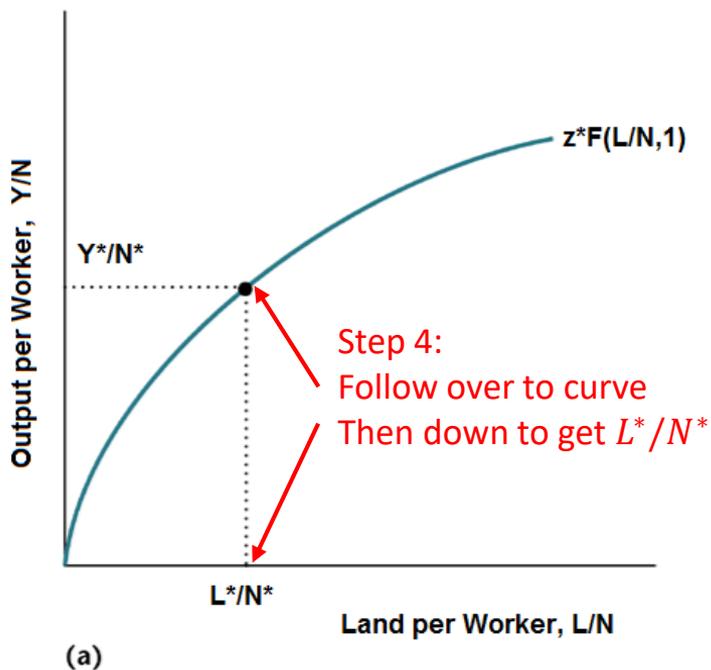
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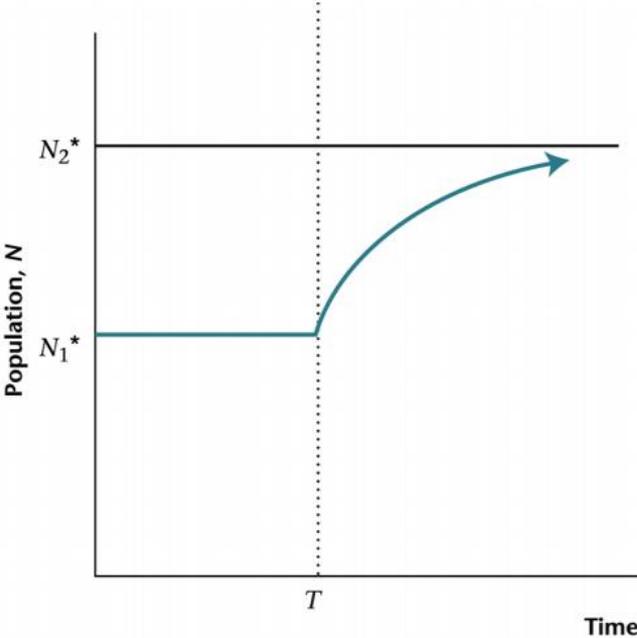
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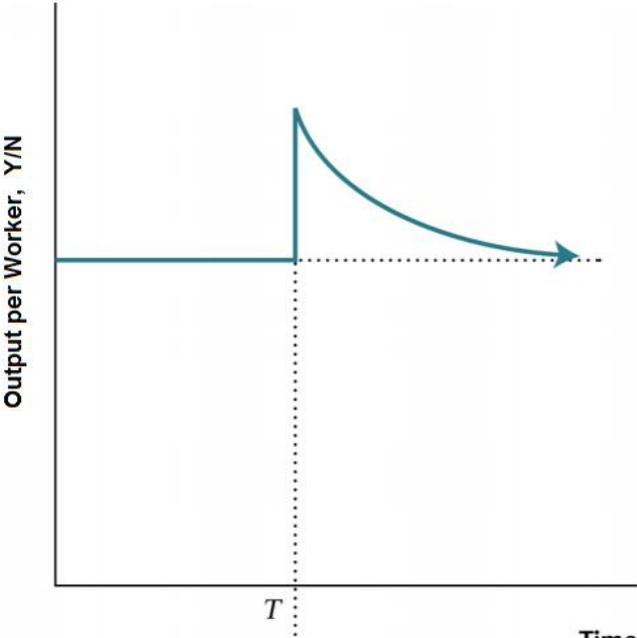
Effect of Increase in TFP on Standard of Living

Suppose we increase z . What happens?

- Only Population Increases. No increase in living standards.



(b)

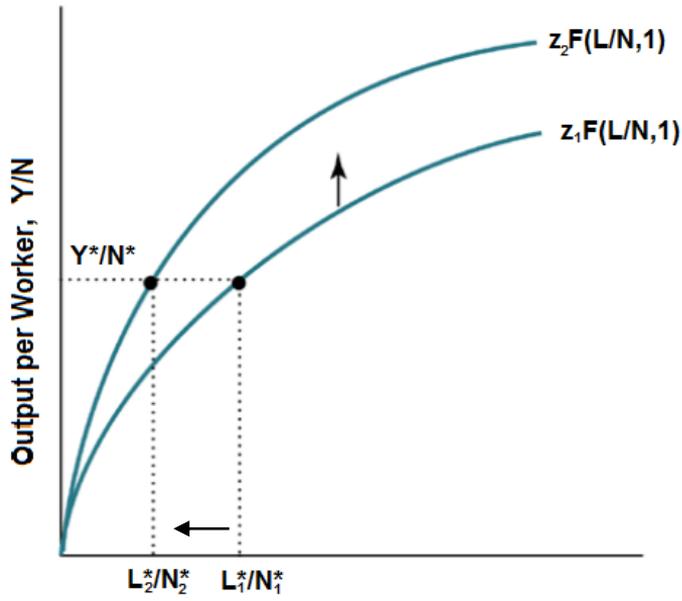


(a)

Effect of Increase in TFP on Standard of Living

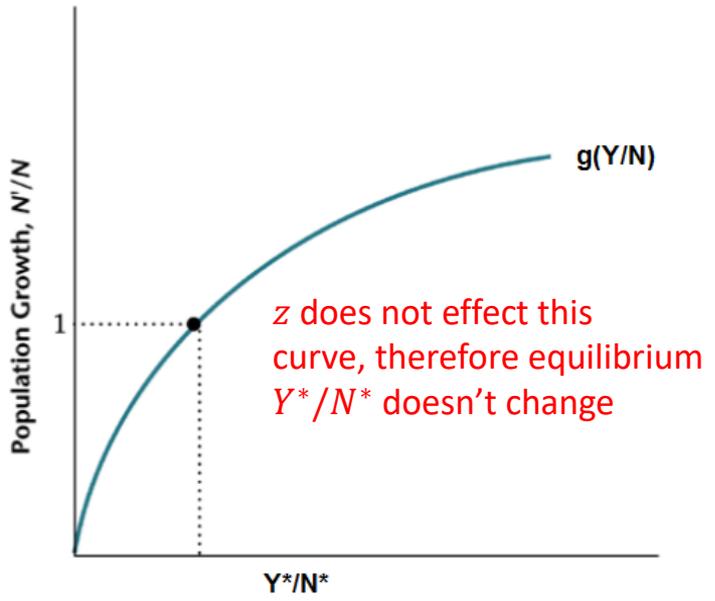
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(a)

Land per Worker, L/N



(b)

z does not effect this curve, therefore equilibrium Y^*/N^* doesn't change

Effect of Increase in TFP on Standard of Living

What is really going on?

- TFP increases output per worker
- This leads people to have more children
- As population increases, Land per Worker decreases
- The above decreases output per worker back to original standards

Policy Implications

In the Malthusian model the only way to increase living standards is population control

- This prevents Land per worker from decreasing
- Keeps output per worker high

Malthusian Model in Practice

When published, the Malthusian model seemed to be an accurate representation

- After the Industrial Revolution, there were immense increases in GDP per capita

What changed?

- Capital became more important (versus Land in fixed supply)
- People voluntarily limited their population growth rates
- As economy develops, opportunity cost of raising large family becomes relatively larger