ECON 256: Poverty, Growth & Inequality

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Growth Accounting

Decompose Output per Worker into three components

1. Productivity
2. Capital-Output Ratio
3. Labor Supplied per Worker

Production Function

Standard aggregate production function is

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

Rewrite as (Same equation, just rewritten. See notes)

$$\frac{Y_t}{N_t} = (A_t)^{\frac{1}{1-\alpha}} \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} \frac{L_t}{N_t}$$

Components: Output

$$\frac{Y_t}{N_t} = (A_t)^{\frac{1}{1-\alpha}} \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} \frac{L_t}{N_t}$$

- Y_t/N_t, Output per Worker.
- Y_t is output or GDP
- N_t is Working Age Population (Population ages 15-64)
- GDP per capita doesn't adjust for portion of population that shouldn't be expected to work. GDP per WAP (Working Age Person) does.

Components: Productivity

$$\frac{Y_t}{N_t} = (A_t)^{\frac{1}{1-\alpha}} \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} \frac{L_t}{N_t}$$

- $(A_t)^{\frac{1}{1-\alpha}}$, Productivity Component
- A_t is **Total Factor Productivity** or TFP
- Embodies things such as technology, education, efficiency, etc.
- Will discuss this more later, including endogenizing it.

Components: Capital

$$\frac{Y_t}{N_t} = (A_t)^{\frac{1}{1-\alpha}} \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} \frac{L_t}{N_t}$$

- $(K_t/Y_t)^{\frac{\alpha}{1-\alpha}}$, Capital component
- K_t/Y_t is Capital-Output Ratio. K_t is Capital Stock.
- Don’t actually observe Capital Stock in National Accounts, only Investment.
- Have to estimate Capital

Components: Labor

$$\frac{Y_t}{N_t} = (A_t)^{\frac{1}{1-\alpha}} \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} \frac{L_t}{N_t}$$

- L_t/N_t, Labor Supply component.
- Ratio of Hours worked, L_t, to Working Age Pop, N_t.
- L_t doesn't adjust for education/skill. That is absorbed in A_t (Can extend model, separate human capital and TFP)

Estimating Capital Stock

- Lack data on Capital stock (total value of all buildings, machines, value of patents, computers, etc)
- Have data on Investment
- Use Law of Motion of Capital

$$K_{t+1} = (1-\delta)K_t + I_t$$

Estimating Capital Stock

- Lack data on Capital stock (total value of all buildings, machines, value of patents, computers, etc)

- Have data on Investment

- Use Law of Motion of Capital

$$K_{t+1} = (1 - \delta)K_t + I_t$$

- **Requires an initial guess $\boldsymbol{K_0}$**

How to Guess Capital Stock?

Most methods give similar results

- Don’t need to guess exactly right, just need it reasonable
- Eventually most of initial capital stock depreciated away and replaced by new investment

Method for Guessing Initial Capital Stock

Set Capital-Output Ratio in first year to average Capital-Output Ratio over next 10 years

$$\frac{K_0}{Y_0} = \frac{1}{10}\sum_{t=1}^{10}\frac{K_t}{Y_t}$$

- Changing K_0 changes $K_1, K_2, \ldots$ so need nonlinear solver to find value. In Excel, can use Goal Seek (see notes):

Goal Seek: Choose $\frac{K_0}{Y_0}$ such that $\left|\frac{K_0}{Y_0} - \frac{1}{10}\sum_{t=1}^{10}\frac{K_t}{Y_t}\right| = 0$

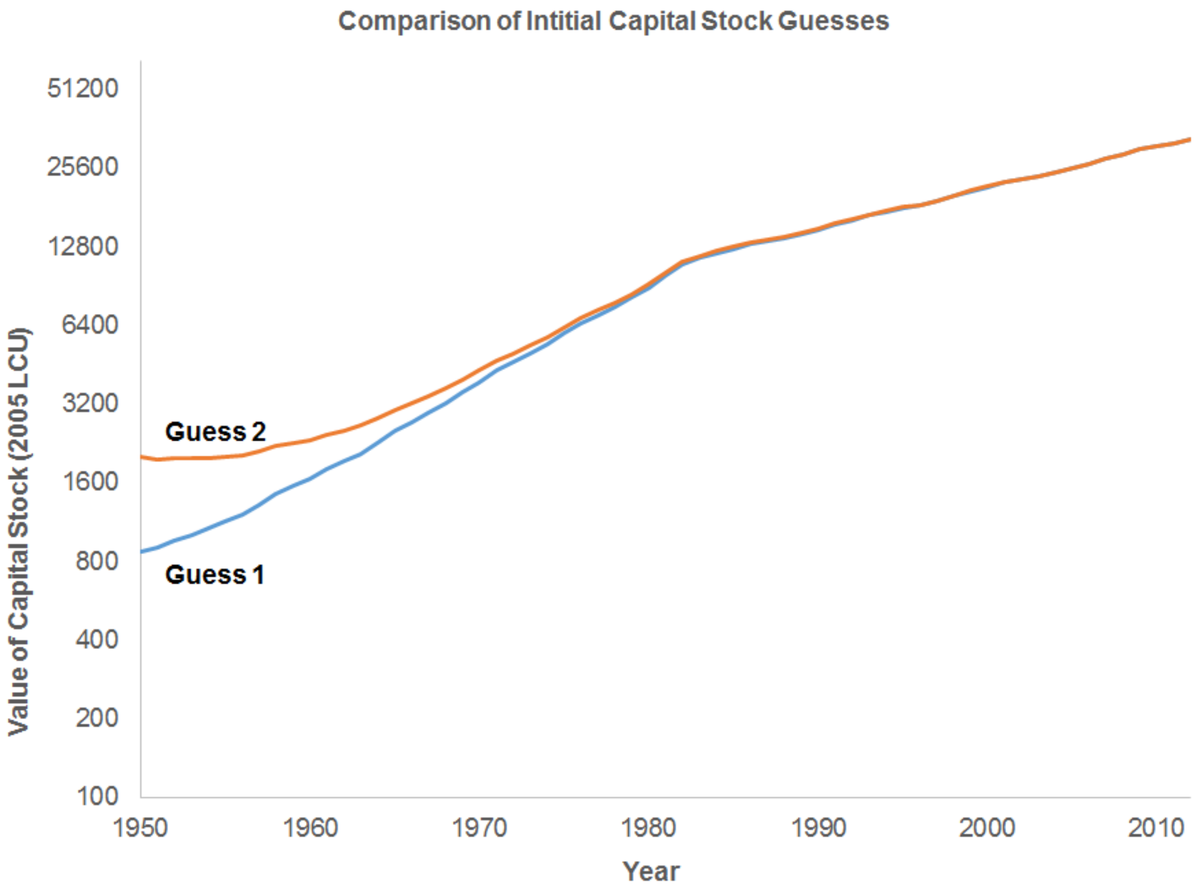
Sensitivity of Capital Stock to Initial Guess

Accuracy of initial guess important for period immediately following your guess

- As time increases, sensitivity decreases. Eventually, guess has negligible impact on estimated capital stock

- Works best when countries have something resembling balanced growth

Sensitivity of Capital Stock to Initial Guess



Depreciation Rate for Growth Accounting

Need to know δ when using Law of Motion of Capital

- National Accounts: Consumption of Fixed Capital $= \delta K_t$
- If know K_t then know δ. Need to jointly estimate δ and K_0

When we lack data, we can use $\delta = 0.05$, which is close to estimated value.

Labor Share of Income

When doing Growth Accounting need to know α

- $\alpha \equiv$Capital Share of Output
- $(1 - \alpha) \equiv$Labor Share of Output

Together they sum to GDP

- In model GDP $=$Output$=$ Income

Estimating α

Estimate $(1-\alpha)$ as Labor Income divided by sum of Capital Income + Labor Income

- Don't count ambiguous income when estimating $(1-\alpha)$

$$(1-\alpha) = \frac{\text{Compesation of Employees}}{\text{GDP} \; - \text{Ambiguous Income}}$$

- For Problem Set 1 use $\alpha = 0.3$; $(1-\alpha) = 0.7$, which are close to estimated values

Estimating TFP

We don't observe A_t directly in the data

- It's a residual. The part of GDP not explained by Capital and Labor alone
- Estimate using the production function

$$A_t = \frac{Y_t}{K_t^{\alpha} L_t^{1-\alpha}}$$

Reminder: Adjusting for Inflation

It’s important to use Real GDP (adjusted for Inflation) and Real Investment when doing growth accounting

- The production function doesn’t have prices in it
- The growth decomposition will be wrong if we have inflation distorting the relative values
- Real GDP often referred to as GDP in constant Prices. That means they compute values evaluated in prices from a different year.

Adjusting for Inflation: Example

Example with Made up Numbers:

- Output in 1950: 10 million apples at $0.10 Each.
- Output in 2000: 30 million apples at $1.00 Each

Nominal GDP (current prices): $1 million in 1950; $30 million in 2000

Real GDP (constant 1950 prices): $1 million in 1950; $3 million in 2000 ($3 million = 30 million apples×$0.10 each)

Growth Accounting

Have all the Variables

- Y_t = Real GDP
- K_t = Real Capital Stock
- L_t = Hours Worked
- N_t = Working Age Population (Pop, Ages 15-64)

Can plug into growth accounting expression and get each component

$$\frac{Y_t}{N_t} = (A_t)^{\frac{1}{1-\alpha}} \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} \frac{L_t}{N_t}$$

Indexing

Interested in Changes over Time

- Units not that important for Growth Accounting

- We index each variable so it is equal to some value in a base year

$$\text{Indexed Value} = 100 \times \left(\frac{\text{Value}}{\text{Value in Base Year}}\right)$$

Indexing Example

Set a base year of 1995

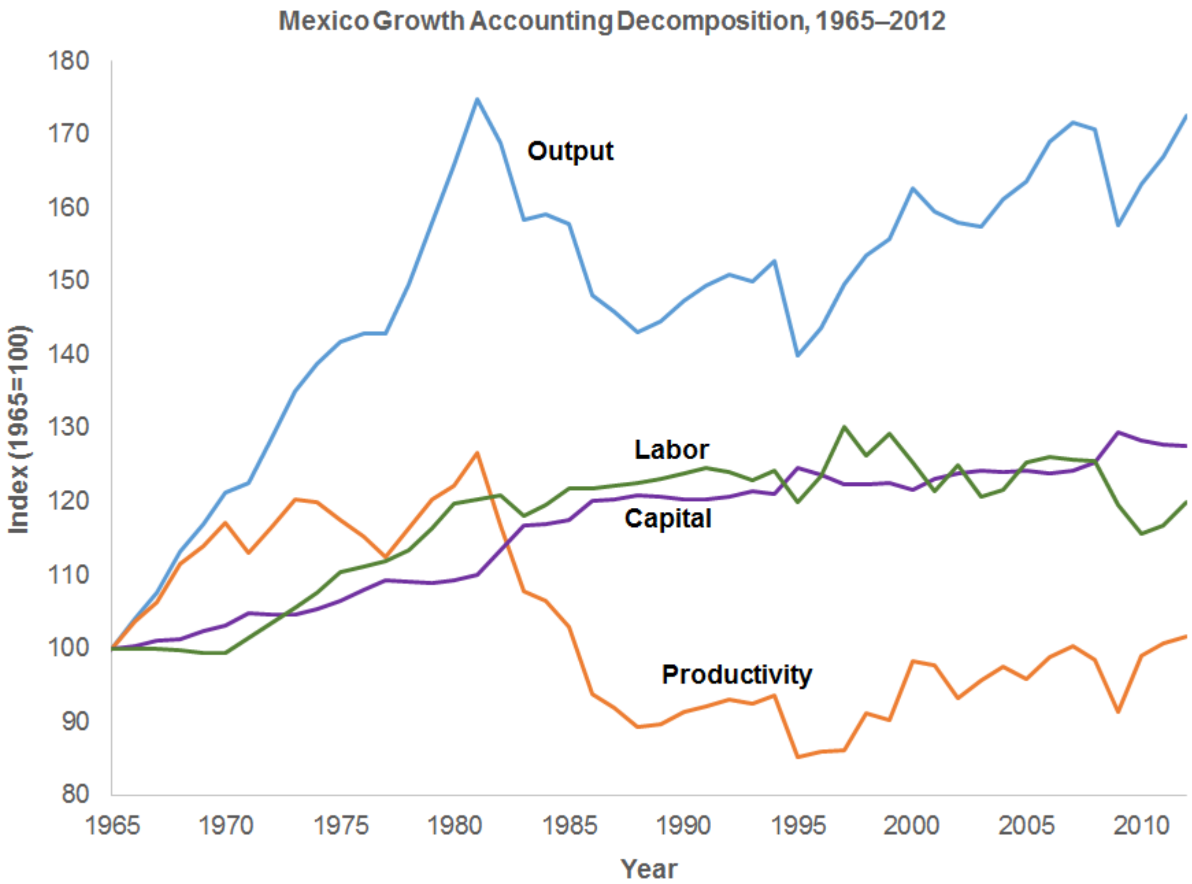
- Value in 1995 = 500; Value in 2000 = 700

$$\text{1995 Indexed Value} = 100 \times \left(\frac{500}{500}\right) = 100$$

$$\text{2000 Indexed Value} = 100 \times \left(\frac{700}{500}\right) = 140$$

- Indicates 2000 value is 40 percent higher than 1995 value

Examples of Growth Accounting



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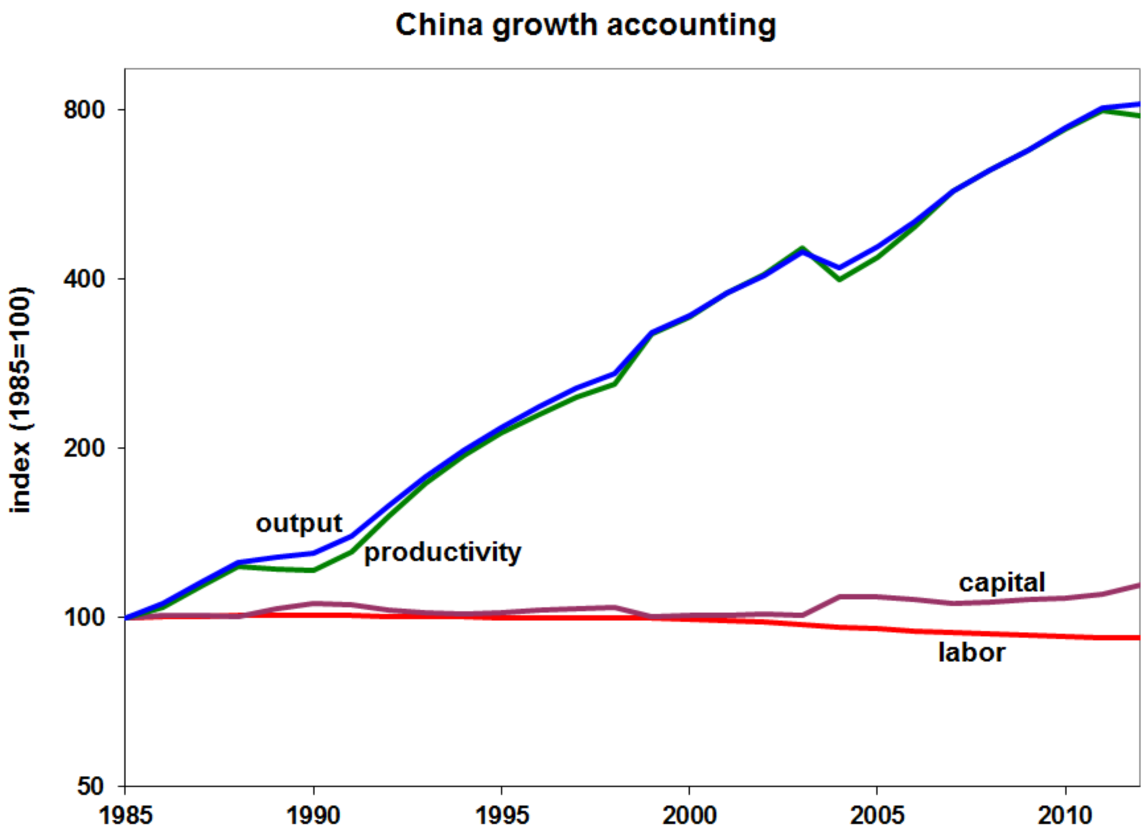

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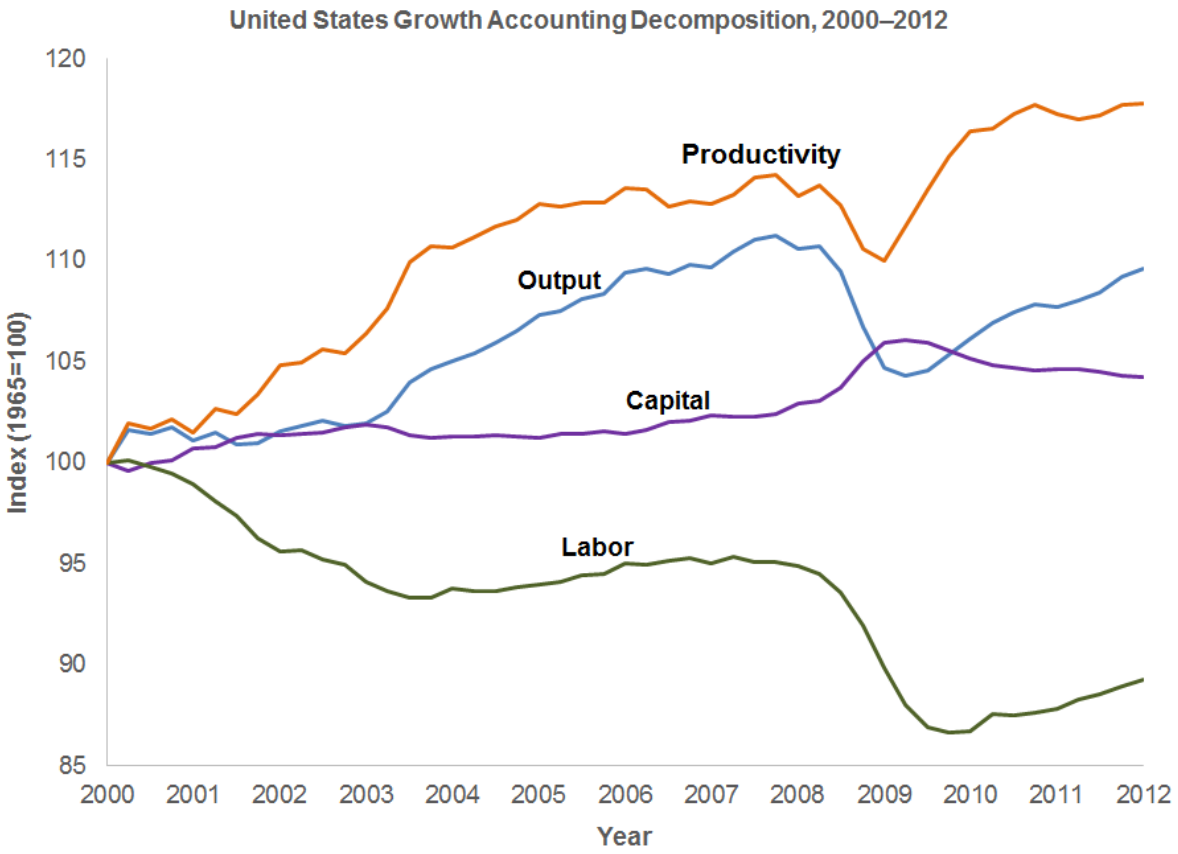

What We Learn From Growth Accounting

- When countries grow, it tends to be through productivity
- If productivity is reason for growth, need a theory of TFP to think about whether countries should converge
- In short run (Business Cycles), other components can be important for explaining fluctuations