The Global Economy

The Production Function

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Roadmap

- Questions to answer

- Production function theory

  - Capital and labor inputs

  - Productivity

- Growth accounting: quantitative analysis

  - Use theory to understand data

 - Decompose sources of output growth

Understanding emerging markets

- Why do some countries grow quickly?
  - From accumulating capital?
  - From productivity growth?
  - How can we measure this?
- How can we forecast long-run growth?
  - How big will China's market be in 20 years?
  - Next week's work…

Why theory?

- Theory is a tool to help organize thoughts

- Theory helps simplify a complex world

- Theory is a framework for analyzing data

Theory: the moving parts
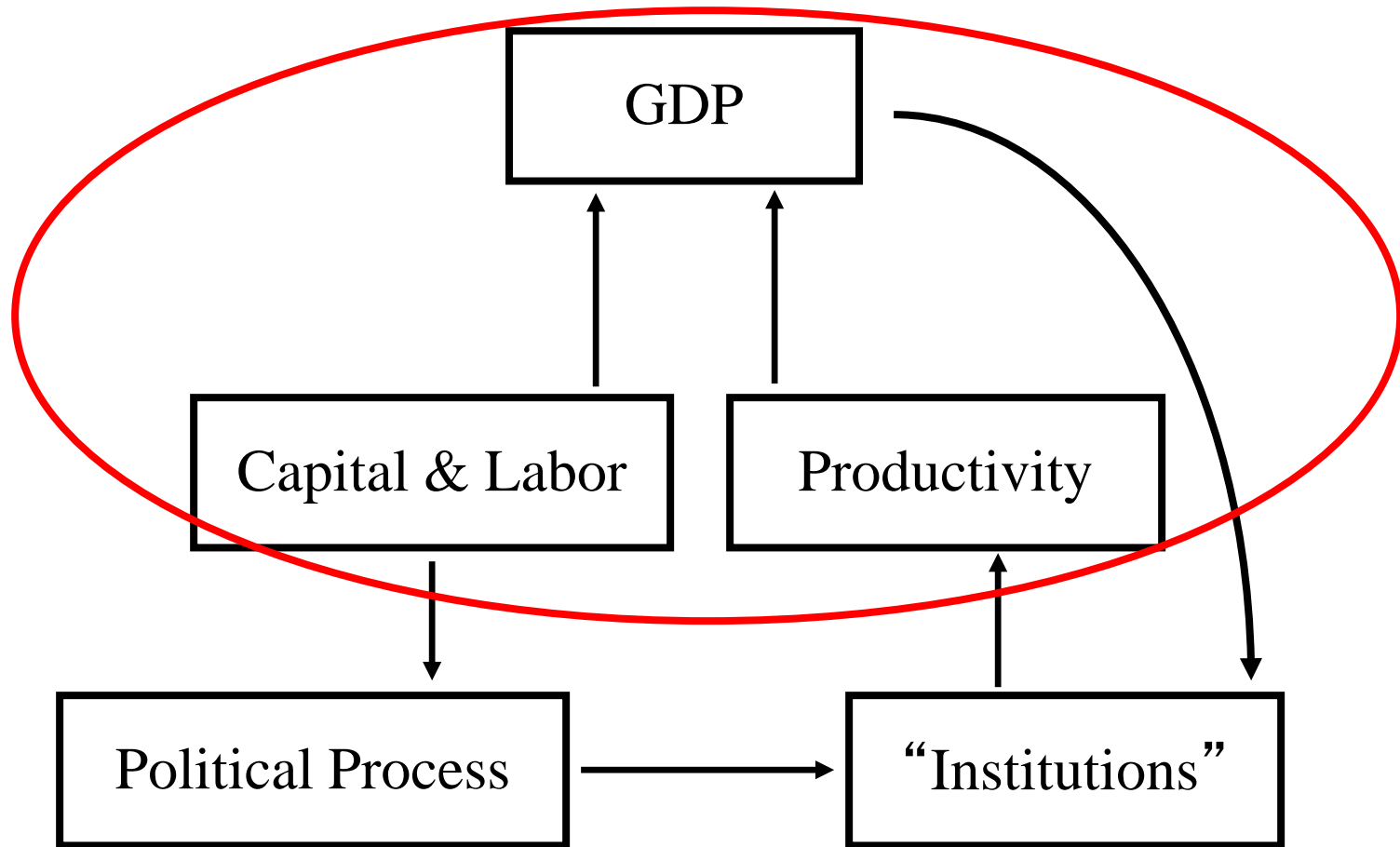
Production function

- Idea: relate output to inputs
- Mathematical version:

$$Y = AK^{\alpha}L^{1-\alpha}$$

- Definitions:
  - K = quantity of physical capital used in production (plant and equipment)
  - L = quantity of labor used in production
  - A = total factor productivity (everything else)

Production function properties

- More inputs lead to more output

  - Positive marginal products of capital and labor

- Diminishing marginal products

  - If we increase one input, each increase leads to less additional output

  - Marginal product = partial derivative of production function

- Constant returns to scale
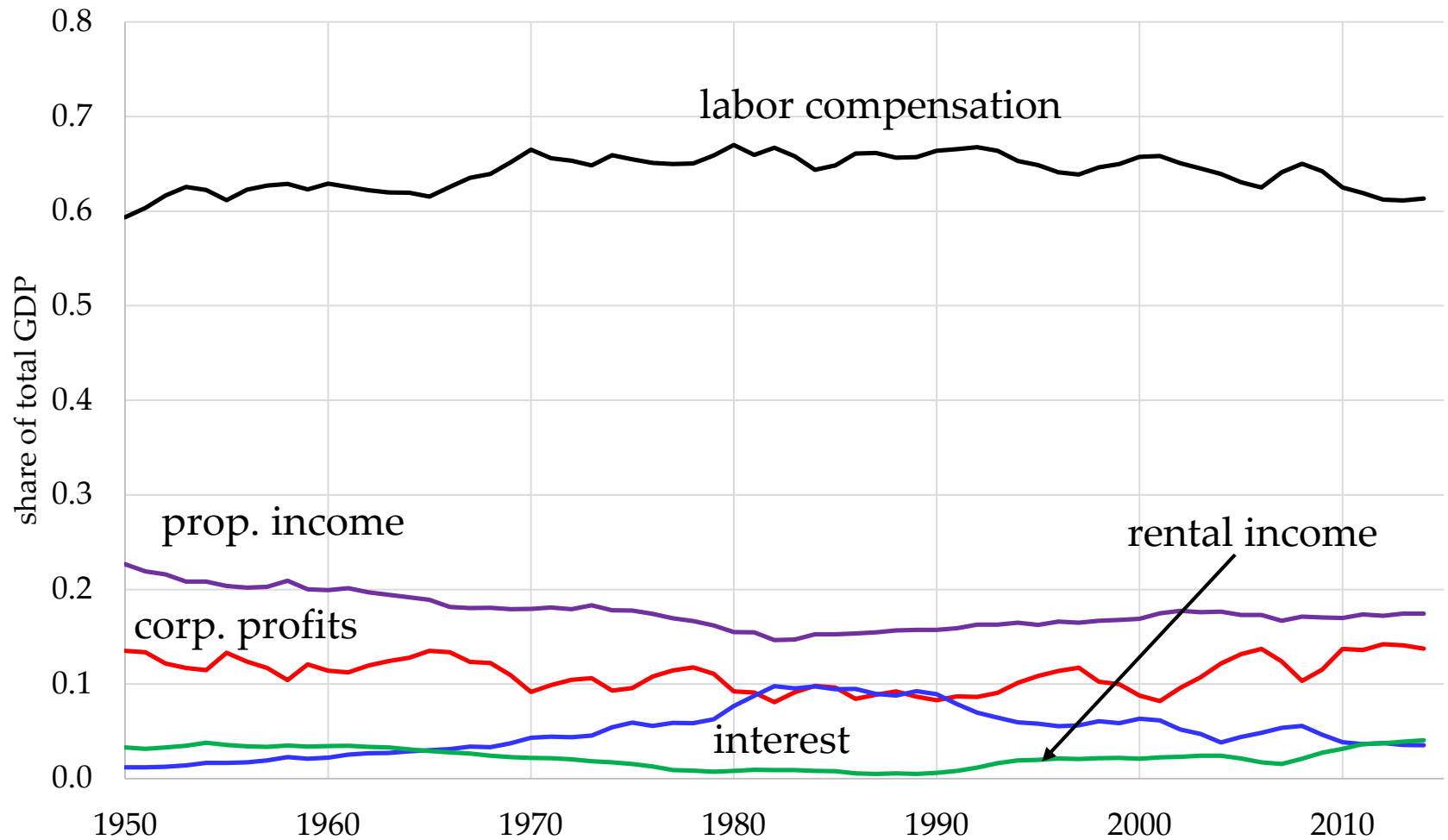
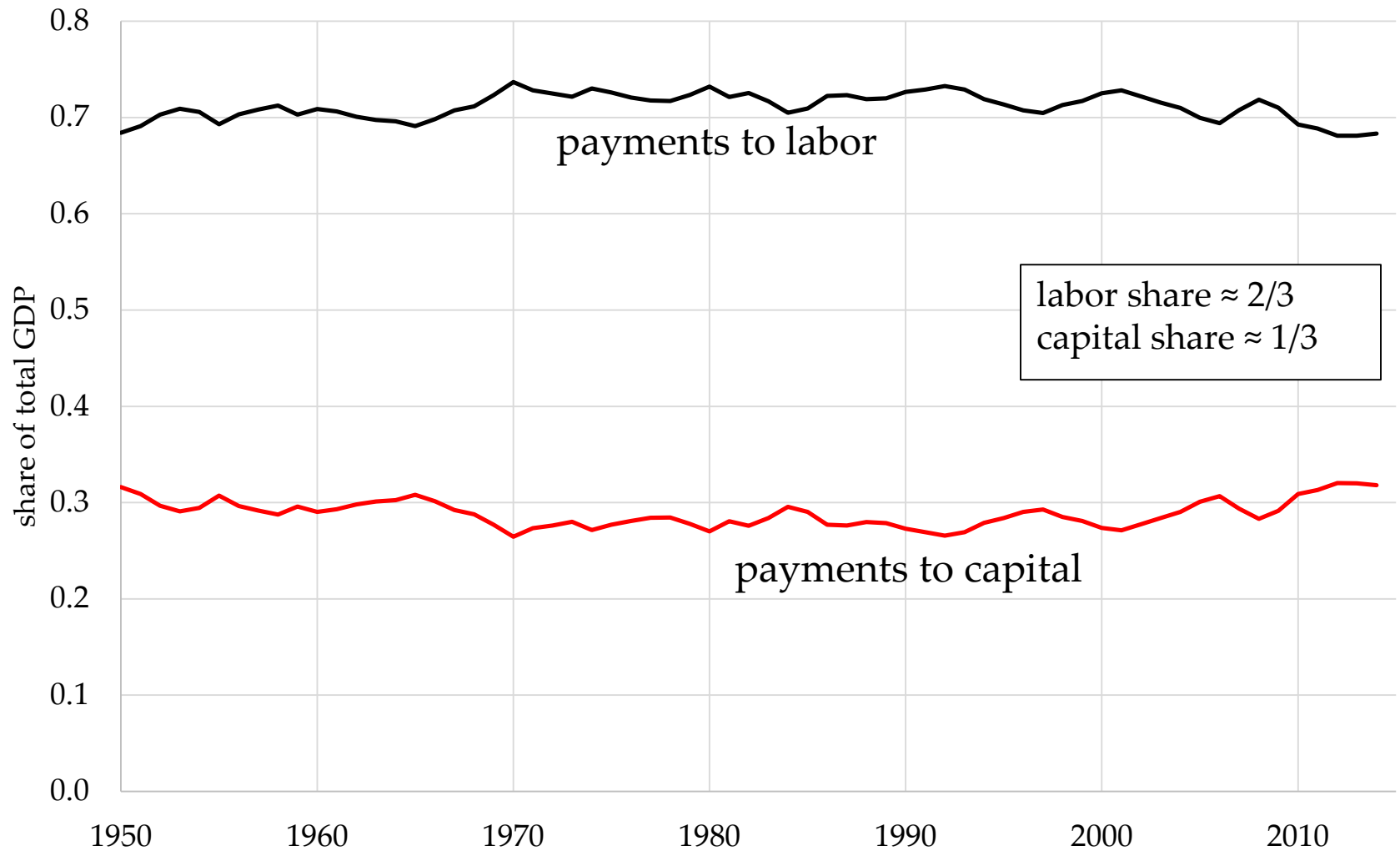 - If we double **both** inputs, we double output

Hold fixed L, increase K

A=1
L=100
$\alpha=1/3$
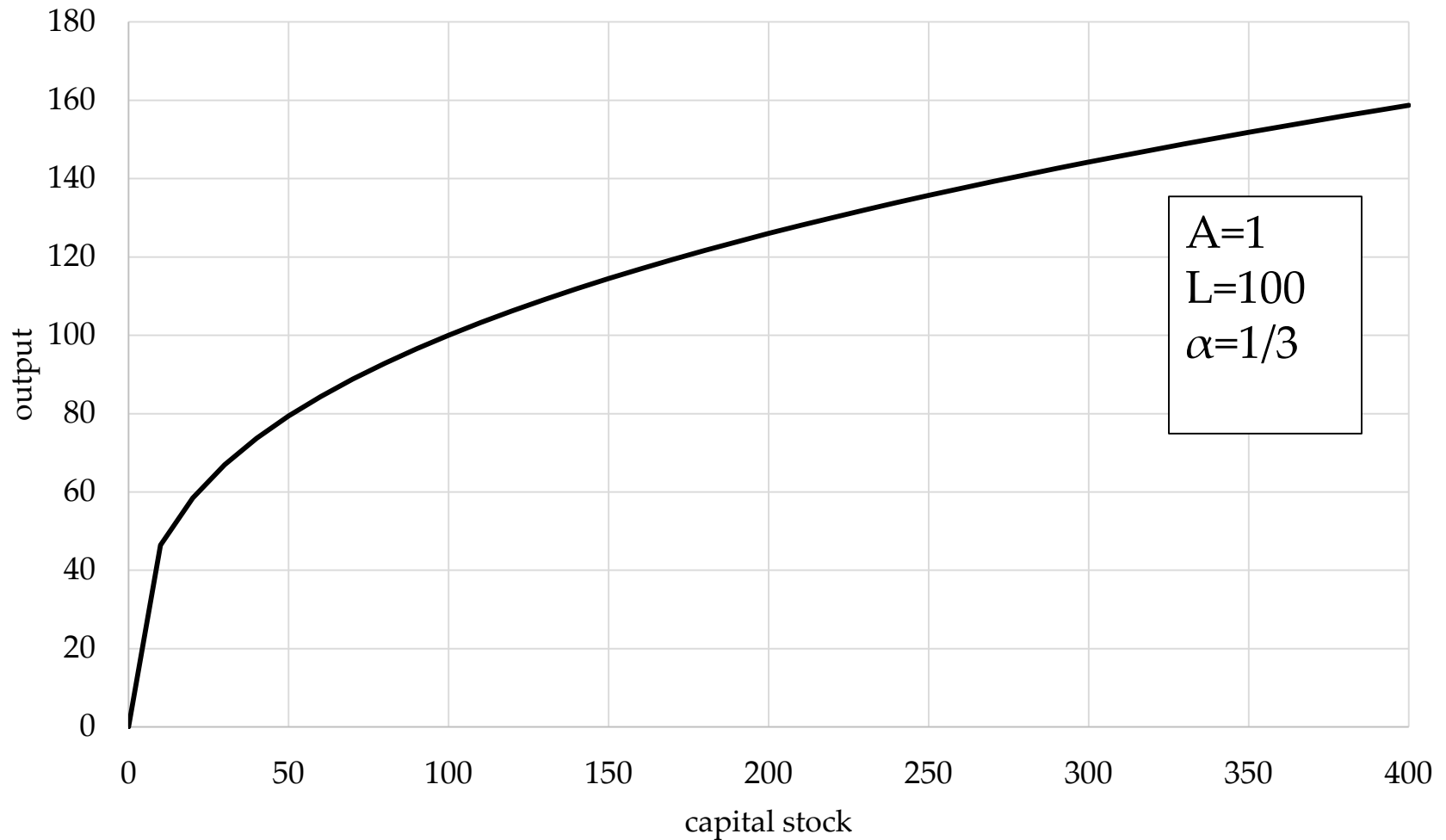
What is α?

- Properties of the Cobb-Douglas function
  - Share of income paid to capital = α
  - Share of income paid to labor = $1 - \alpha$
  - These share are constant, regardless of output level
- Income shares are (roughly) constant in data
  - Share of income to capital = α = 1/3

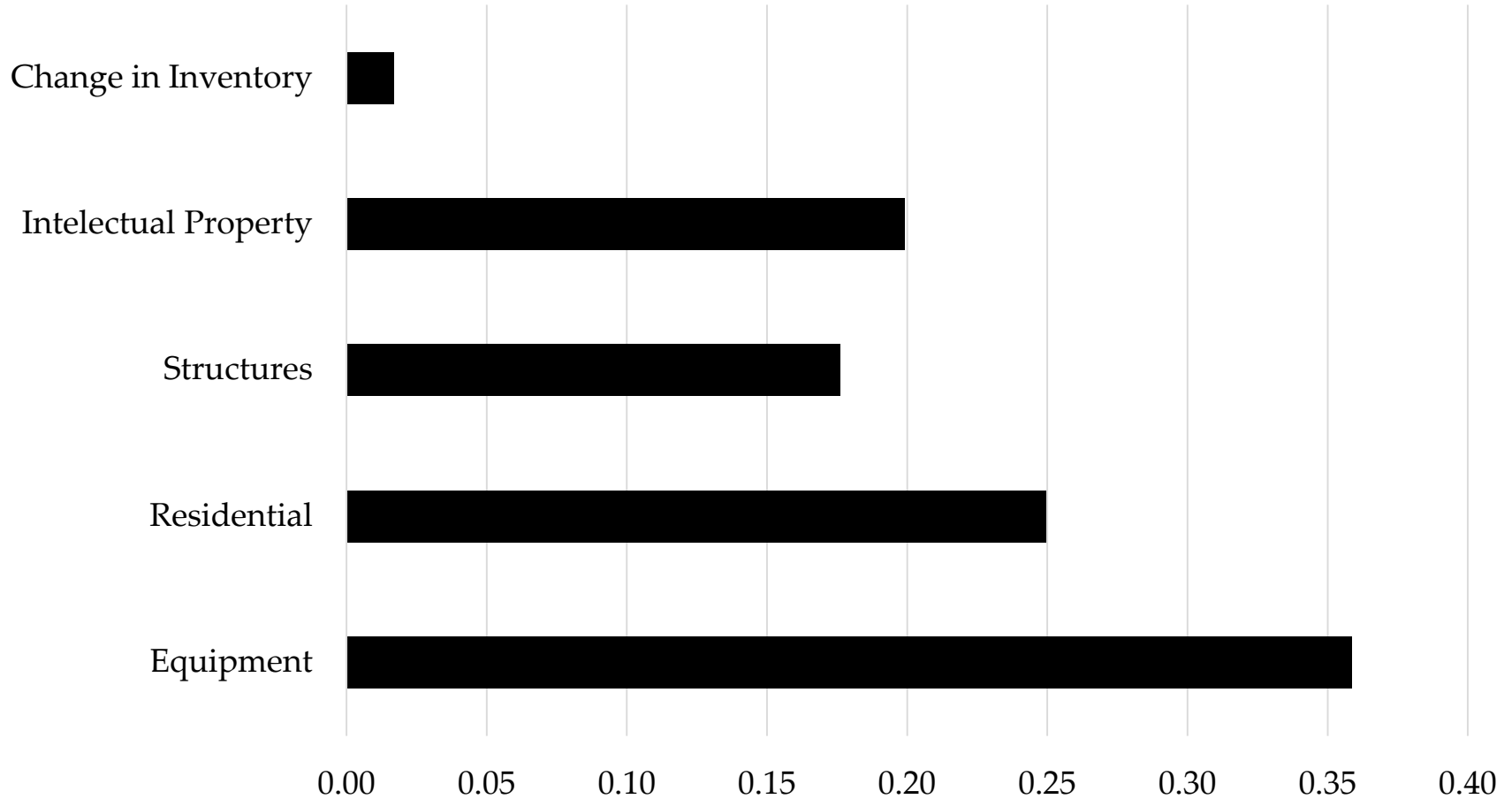# GDP by income type

GDP by income type

Inputs: capital

- Meaning: physical capital, plant and equipment

- Why does it change?

  - Depreciation/destruction

  - New investment

- Mathematical version:

$$K_{t+1} = K_t - \delta K_t + I_t$$
$$= (1 - \delta)K_t + I_t$$

- Adjustments for quality?

Investment composition

The return to capital

- Real return = interest rate – depreciation

$$r = \alpha \times Y / K - \delta$$

- In the United States: $Y / K = 0.5, \delta = 0.10$

$$r = 0.33 \times 0.5 - 0.10 = 0.065$$

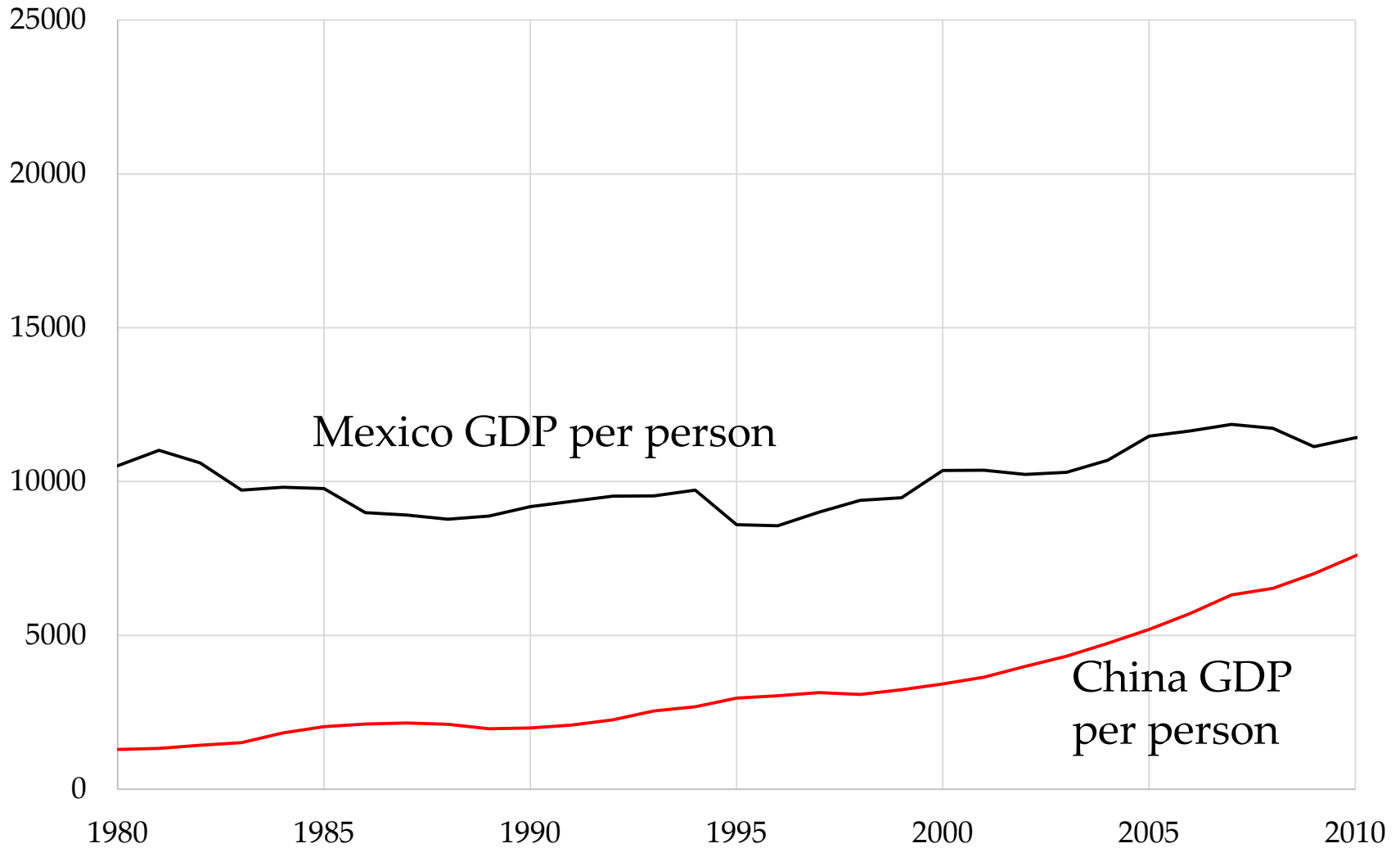- As Y/K decreases, so does the return

Inputs: labor

- Meaning: units of work effort

- Why does it change?
  - Population growth and demographics
  - Fraction of population employed
  - Hours worked per worker

- Adjustments for quality?
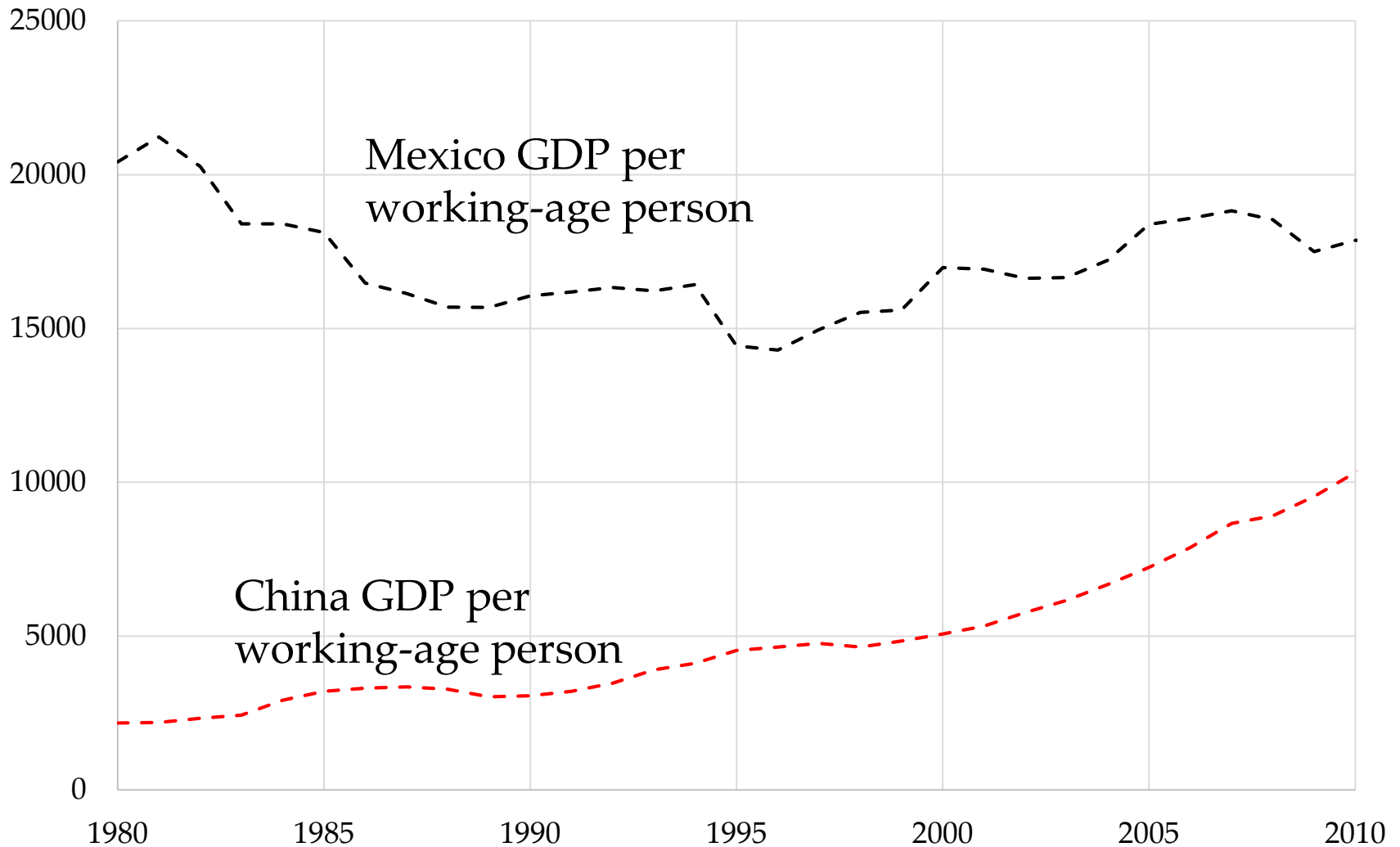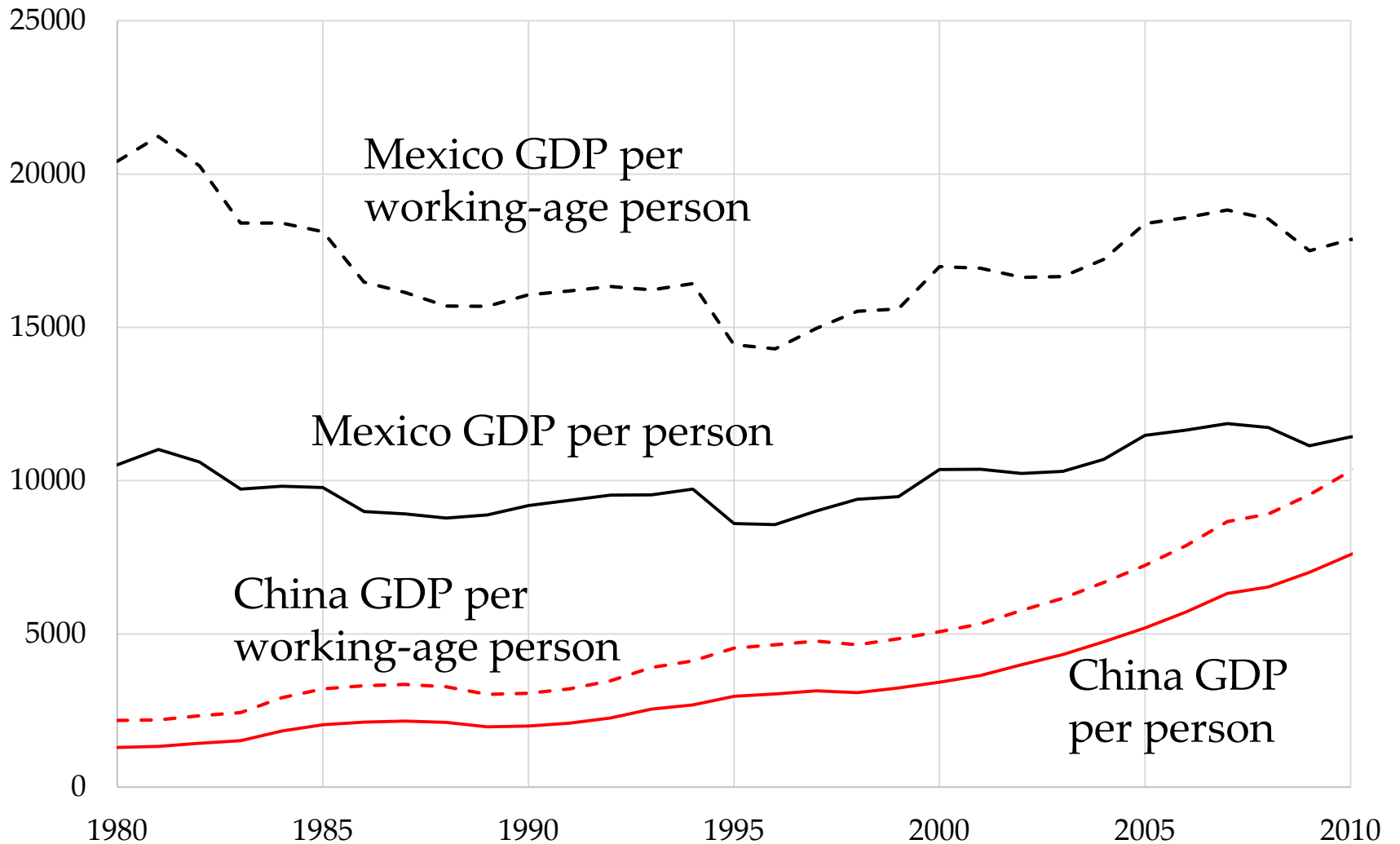 - Human capital: education, experience
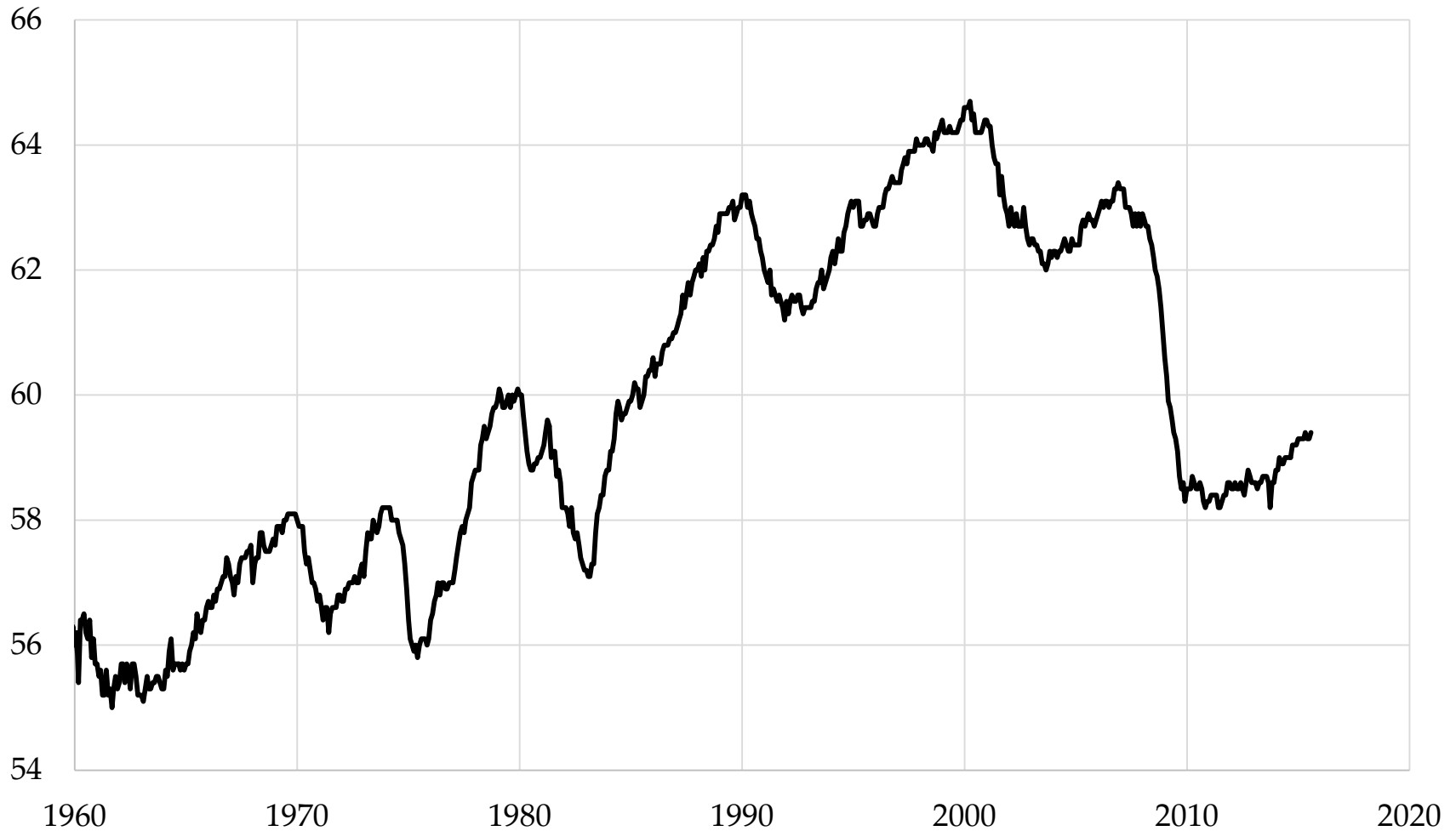
Age distribution

GDP per capita

Mexico GDP per person

China GDP per person
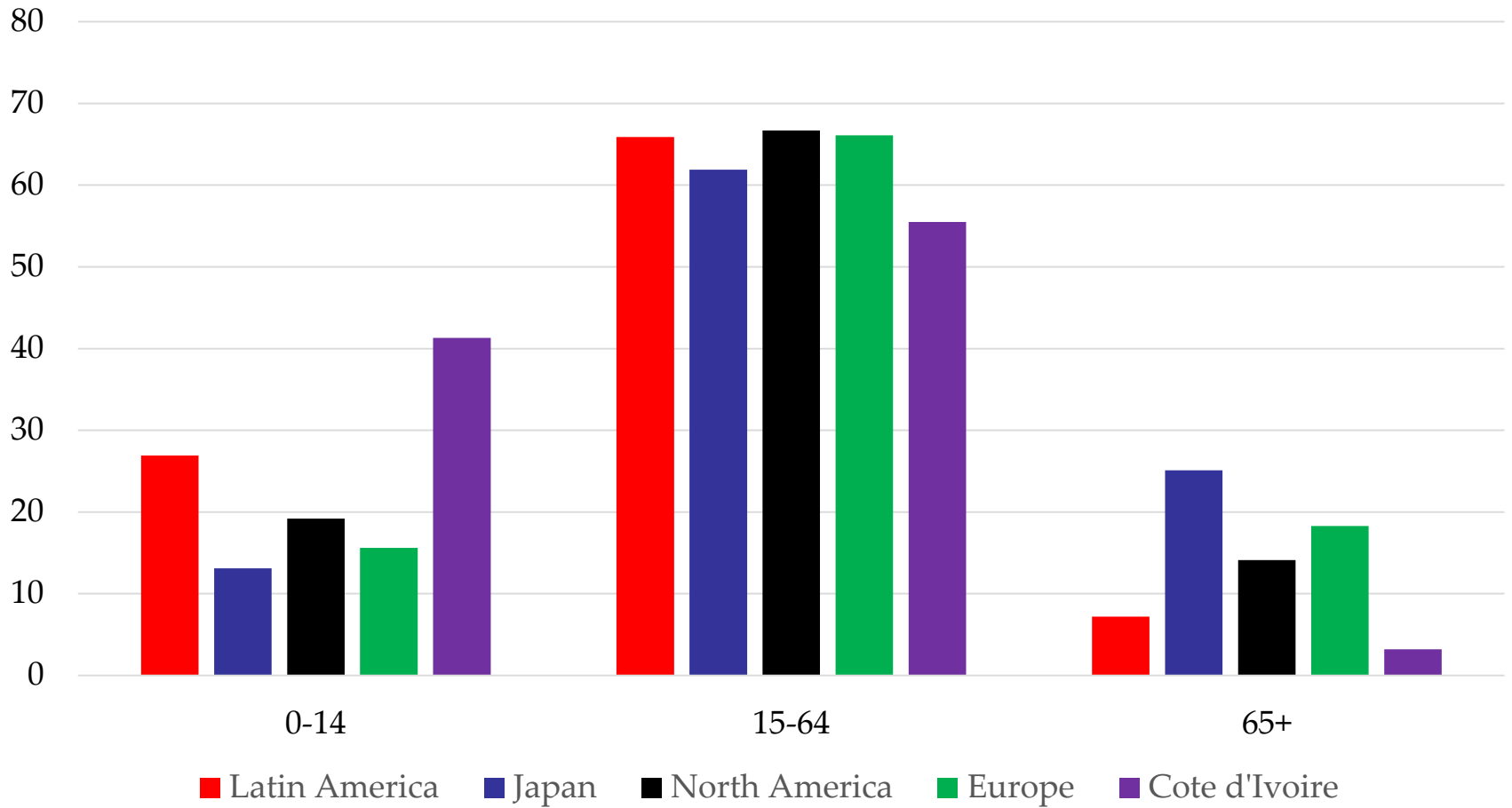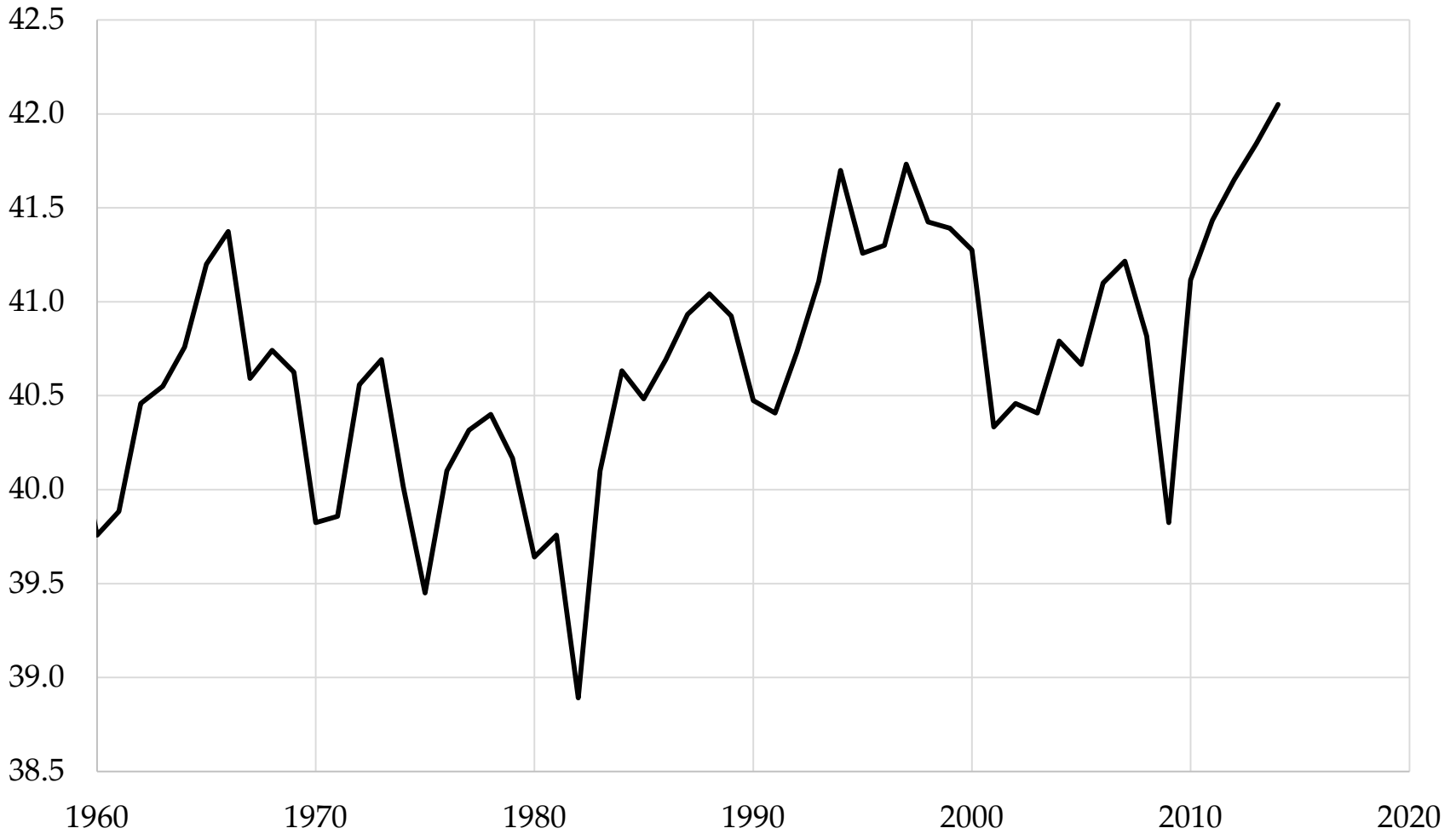
GDP per working-age person

GDP per working-age person

Employment-population ratio

Weekly hours in manufacturing

Productivity

- Standard number:

  - Average product of labor: Y/L

- Our number:

  - Total Factor Productivity: $Y/(K^{\alpha}L^{1-\alpha})$

- How do we measure it?

  - Solve the production function for A

$$Y = AK^{\alpha}L^{1-\alpha}$$

$$A = \frac{Y}{K^{\alpha}L^{1-\alpha}}$$

Productivity

$$A = \frac{Y}{K^{\alpha}L^{1-\alpha}} = \frac{Y}{L} \times \left(\frac{K}{L}\right)^{-\alpha}$$

- Example (US): Y/L = 33, K/L = 65

$$A = 33 \times 65^{-0.33} = 8.21$$

- Note: the TFP number by itself is meaningless, but comparisons across countries or time are useful.

Production function review

$$Y = AK^{\alpha}L^{1-\alpha}$$

- What changes **in this equation** if

  - **A firm builds a new factory?**

  - Fewer people retire at 65?

  - **Spanish banks channel funds to unproductive firms?**

  - Workers shift from subsistence farming to industry in Viet Nam?

  - **Competition drives inefficient firms out of business?**

  - Venture capital funds identify good unfunded projects?

  - **Alaska builds a bridge to nowhere?**

  - China invests in massive infrastructure projects?

Production function takeaways

- The production function links output to inputs and productivity:

$$Y = AK^{\alpha}L^{1-\alpha}$$

- The capital input (K):

  – Plant and equipment, a consequence of investment (I)

- The labor input (L):

  – Population growth, age distribution, participation and hours

- TFP (A) is computed from data on output and inputs

  – TFP is "everything else"