

# ECO 442: Quantitative Trade Models

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# Trade Costs and Tariffs

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So far in our models, international trade has been frictionless

- Instantly teleport goods from one country to another at no cost
- Obviously not true in the real world

Focus on two primary trade costs

- **Iceberg Trade Costs**
- **Tariffs**

# Iceberg Trade Costs

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Iceberg Trade Costs are costs associated with transporting goods across countries

- Fuel to ship the goods
- Loss of product due to spoilage
- Additional workers needed to fill out paper work and follow international regulations

Iceberg trade costs means to deliver 1 unit of exports, necessary to ship  $\tau > 1$  units

- For simplicity, we set domestic iceberg trade costs as  $\tau = 1$

# Tariffs

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Tariffs are a tax imposed on imports

- Tariffs are redistributed to consumers in the country imposing the tariff

$$\text{Income} = \overbrace{wL}^{\text{Labor Income}} + \overbrace{T}^{\text{Tariff Income}}$$

- Unlike iceberg costs, nothing is physically lost
- Like iceberg costs, the presence of Tariffs distorts the equilibrium vs a frictionless world
- Tariffs are typically ad-valorem (applied proportionally to value). Model as

price with tariff = tariff  $\times$  price without tariff

$$p^{\text{import}} = \tau p^{\text{world}}$$

# Multi-Good Ricardian Model of Trade

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Comparative advantage: countries differ in technology and therefore opportunity costs.

- 2x2 Ricardian Model is simple. Tariffs too high  $\Rightarrow$  stop trading.
- Tariffs more interesting with multiple goods (adds middleground between Trade and Autarky)

Dornbusch, Fischer and Samuelson (1979)

- Ricardian model with 2 countries and a continuum of goods

# Model Setup

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Two countries,  $i, j = 1, 2$

Continuum of goods indexed by  $z \in [0, 1]$

Labor ( $L_i$ ) is only factor of production, supplied inelastically

Countries differ in labor productivities for each good ( $a_i(z)$ ):  $y_i = l_i(z)/a_i(z)$

- Order the goods by relative comparative advantage  $\frac{a_2(z)}{a_1(z)}$  (highest at 0, lowest at 1)
  - Country 1 can make goods relatively cheaper near zero, and country 2 can make goods relatively cheaper near one.

# Equilibrium Definition

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Equilibrium elements

- Prices for each good:  $\hat{p}(z)$
- Wages:  $\hat{w}_1, \hat{w}_2$
- Consumption, labor input, and production for each good:  $\hat{c}_i(z), \hat{y}_i(z), \hat{l}_i(z)$

Such that

- 1) Given prices and wages, consumers maximize utility subject to budget constraint
- 2) Firms maximize profit for each good produced in a country
- 3) Markets clear (consumption = output, labor used in production sums to labor endowment)

# 1. Consumers problem

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Given  $\hat{p}(z)$ ,  $\hat{w}_i$  consumer in country  $i$  to solve

$$\max \int_0^1 U(c_i(z)) dz$$

subject to budget constraint

$$\int_0^1 \hat{p}(z)c_i(z) = \hat{w}_i L_i$$

$$c_i(z) \geq 0, \forall z$$



## 2. Firms Problem

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Given  $\hat{p}(z)$ ,  $\hat{w}_i$  firms in country  $i$  maximize profits for each good  $z$

$$\max \hat{p}(z)y_i(z) - \hat{w}_i l_i(z)$$

subject to production technology:

$$y_i(z) = l_i(z)/a_i(z)$$

Where  $a_i(z)$  is unit labor costs for good  $z$  in country  $i$

**Firm Optimization Yields:**  $\hat{p}(z) = \hat{w}_i a_i(z)$  if  $\hat{y}_i(z) > 0$

### 3. Market Clearing

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Goods market clears for each good (if autarky instead of trade would have  $\hat{c}_i(z) = \hat{y}_i(z)$ )

$$\hat{c}_1(z) + \hat{c}_2(z) = \hat{y}_1(z) + \hat{y}_2(z), \quad \forall z \in [0,1]$$

Labor market clears for each country

$$\int_0^1 \hat{l}_i(z) dz = L_i, \quad i = 1,2$$

## Equilibrium Solution: Cutoff

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Cutoff  $\bar{z}$  such that country 1 produces and exports goods in  $[0, \bar{z}]$

Country 2 produces and exports goods in  $(\bar{z}, 1]$

# Equilibrium Solution: Cutoff

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## Analytic Simplifications:

Cobb Douglas Preferences:  $U(c_i(z)) = \log(c_i(z))$

Symmetry:  $L_1 = L_2 = L$

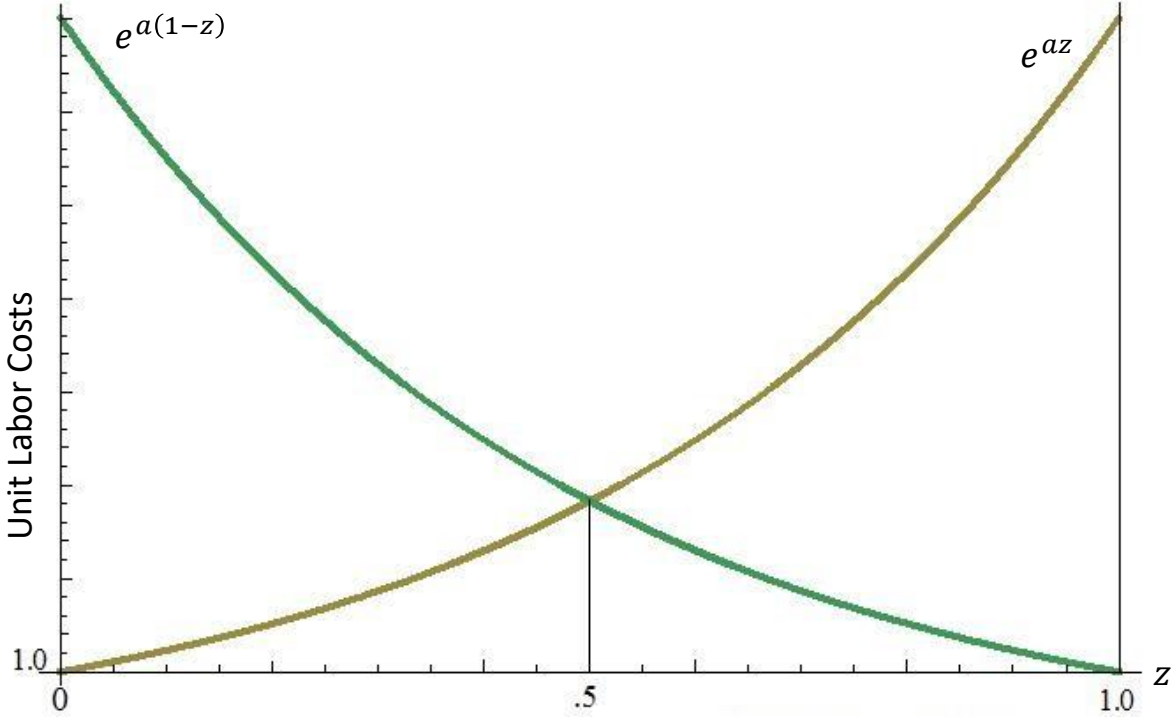
Production technologies:

$$a_1(z) = e^{az}$$

$$a_2(z) = e^{a(1-z)}$$

# Production Technologies

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# Symmetry

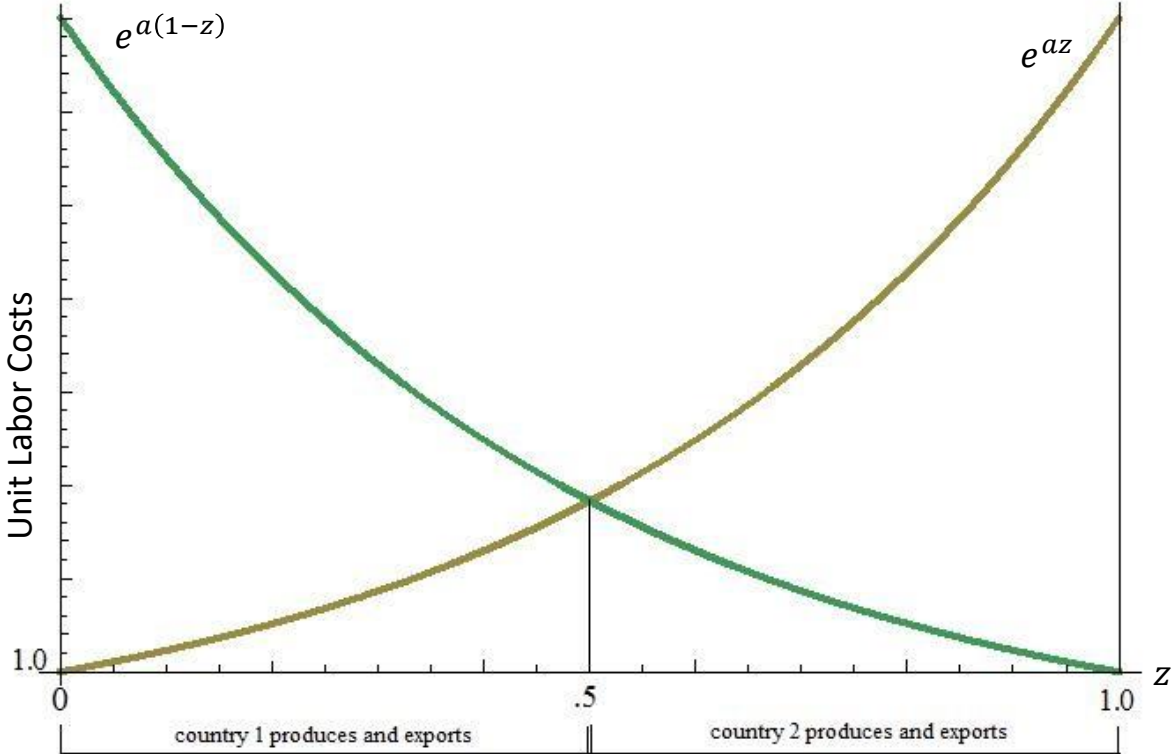
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Will have equilibrium with  $\hat{w}_1 = \hat{w}_2$  (relative wages  $\frac{\hat{w}_1}{\hat{w}_2} = 1$ )

Will have cutoff  $\bar{z} = 1/2$

- Country 1 produces and exports goods in  $Z_1 = \left[0, \frac{1}{2}\right]$
- Country 2 produces and exports goods in  $Z_2 = \left(\frac{1}{2}, 1\right]$

# Symmetric Equilibrium



# Symmetric Equilibrium

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- Prices

$$\hat{p}(z) = \begin{cases} e^{az}, & z \in [0, \bar{z}) \\ e^{a(1-z)}, & z \in (\bar{z}, 1] \end{cases}$$

- Wages:  $\hat{w}_1, \hat{w}_2 = 1$  (normalize wages to 1)
- Consumption, labor input, and production:

$$\hat{c}_i(z) = \frac{L}{\hat{p}(z)} \quad \forall z \in [0,1]$$

$$\hat{y}_i(z) = \frac{2L}{\hat{p}(z)} \text{ if } z \in Z_i, \quad (\hat{y}_i(z) = 0 \text{ if } z \notin Z_i)$$

$$\hat{l}_i(z) = 2L \text{ if } z \in Z_i, \quad (\hat{l}_i(z) = 0 \text{ if } z \notin Z_i)$$



# Trade Costs and Trade Policy

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Have a working model, can use it to think about the effects of trade costs and trade policy

## Iceberg Trade Costs:

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Suppose each country faces an iceberg transportation cost of  $\tau > 1$  when exporting:

$$y_i^j(z) = \frac{l_i(z)}{\tau a_i(z)}, \quad \text{if } i \neq j$$

Still costless to produce for domestic market:  $y_i^i(z) = l_i(z)/a_i(z)$

# Iceberg Trade Costs: Cutoffs

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Will no longer have single cutoff

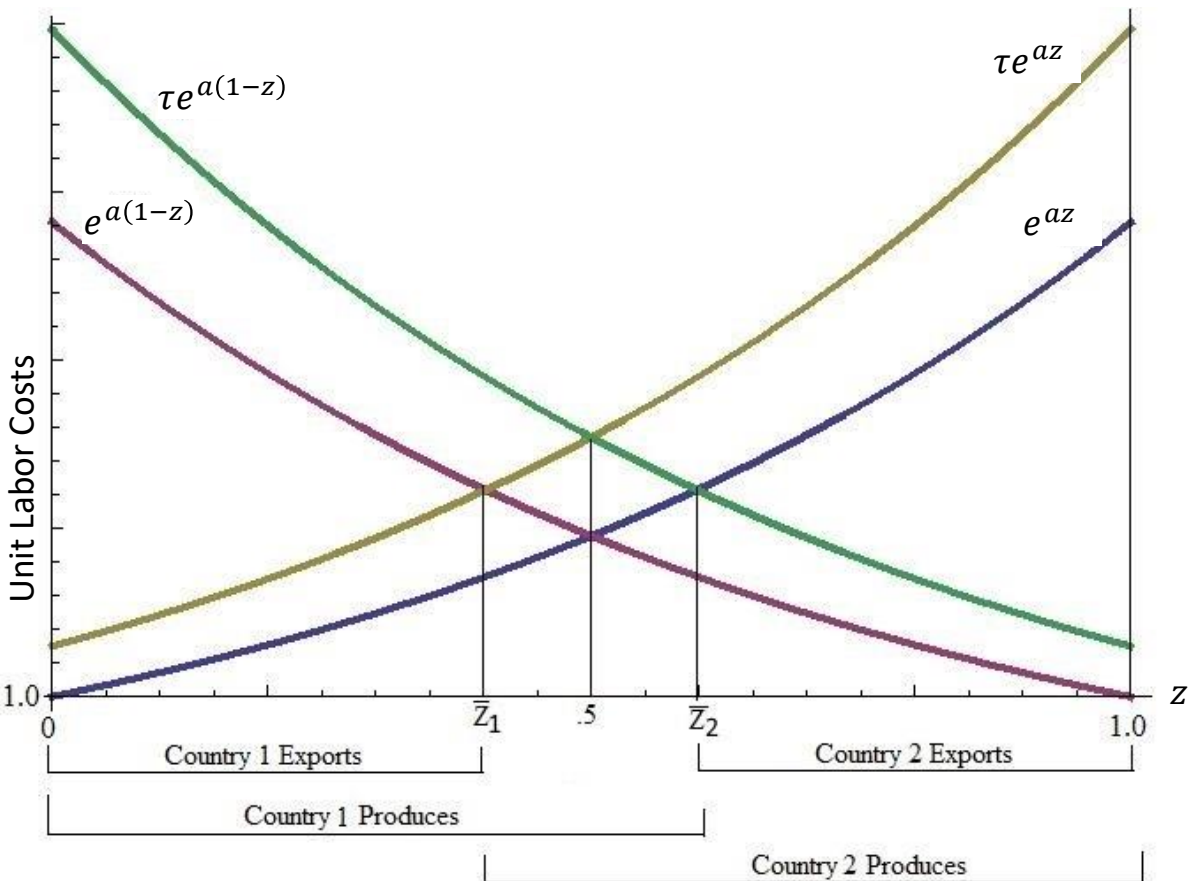
- If  $\tau > \max \left\{ \frac{a_1(z)}{a_2(z)}, \frac{a_2(z)}{a_1(z)} \right\}$  then good will not be exported by either country

Two cutoffs  $\bar{z}_1$  and  $\bar{z}_2$  ( $\bar{z}_2 > \bar{z}_1$ ):

- Country 1 produces goods  $z \in [0, \bar{z}_2]$ , exports goods  $z \in [0, \bar{z}_1]$
- Country 2 produces goods  $z \in (\bar{z}_1, 1]$ , exports goods  $z \in (\bar{z}_2, 1]$

Keep the same symmetric setup, so still have an equilibrium with  $\hat{w}_1 = \hat{w}_2 = 1$ .

# Symmetric Equilibrium: Iceberg Trade Costs



# Tariff Trade Costs

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Now suppose  $\tau$  is an ad valorem tariff instead of an iceberg trade cost.

- Tariffs go on prices rather than in production function
- Let  $p_i(z)$  be the price firms in  $i$  charge domestically if they produce the good
- Let  $p_i^j(z)$  be the price of good  $z$ , produced in country  $i$ , and consumed in country  $j$ , then

$$p_i^j(z) = \begin{cases} \tau p_i(z), & \text{if } i \neq j \\ p_i(z), & \text{if } i = j \end{cases}$$

# Tariff Trade Costs

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For both iceberg trade costs and tariffs, will have

$$\hat{p}_i^j(z)y_i^j(z) = \begin{cases} \frac{\hat{w}_i l_i(z)}{\tau}, & \text{if } i \neq j \\ \hat{w}_i l_i(z), & \text{if } i = j \end{cases}$$

This means it doesn't matter if we put  $\tau$  on prices or output. Solution to problem is same.

Difference is that tariffs are rebated back to consumers. Consumer budget constraint:

$$\int_0^1 \hat{p}_i(z)c_i(z)dz = \hat{w}_i L_i + \overset{\text{Tariff Revenue}}{\hat{T}_i}$$

$$T_i = \int_z (\tau - 1)\hat{p}_j(z)y_j^i(z)dz$$

# Cutoffs and Symmetric Equilibrium: Tariff Trade Costs

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Same as with iceberg trade costs.

Two cutoffs  $\bar{z}_1$  and  $\bar{z}_2$  ( $\bar{z}_2 > \bar{z}_1$ ):

- Country 1 produces goods  $z \in [0, \bar{z}_2]$ , exports goods  $z \in [0, \bar{z}_1]$
- Country 2 produces goods  $z \in (\bar{z}_1, 1]$ , exports goods  $z \in (\bar{z}_2, 1]$

# Consumption Differences: Iceberg Costs vs Tariffs

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Under iceberg transportation costs:

$$c_1(z) = \begin{cases} \frac{L}{e^{az}}, z \in [0, \bar{z}_2] \\ \frac{L}{\tau e^{a(1-z)}}, z \in (\bar{z}_2, 1] \end{cases} ; c_2(z) = \begin{cases} \frac{L}{\tau e^{az}}, z \in [0, \bar{z}_1] \\ \frac{L}{e^{a(1-z)}}, z \in (\bar{z}_1, 0] \end{cases}$$

Under tariffs

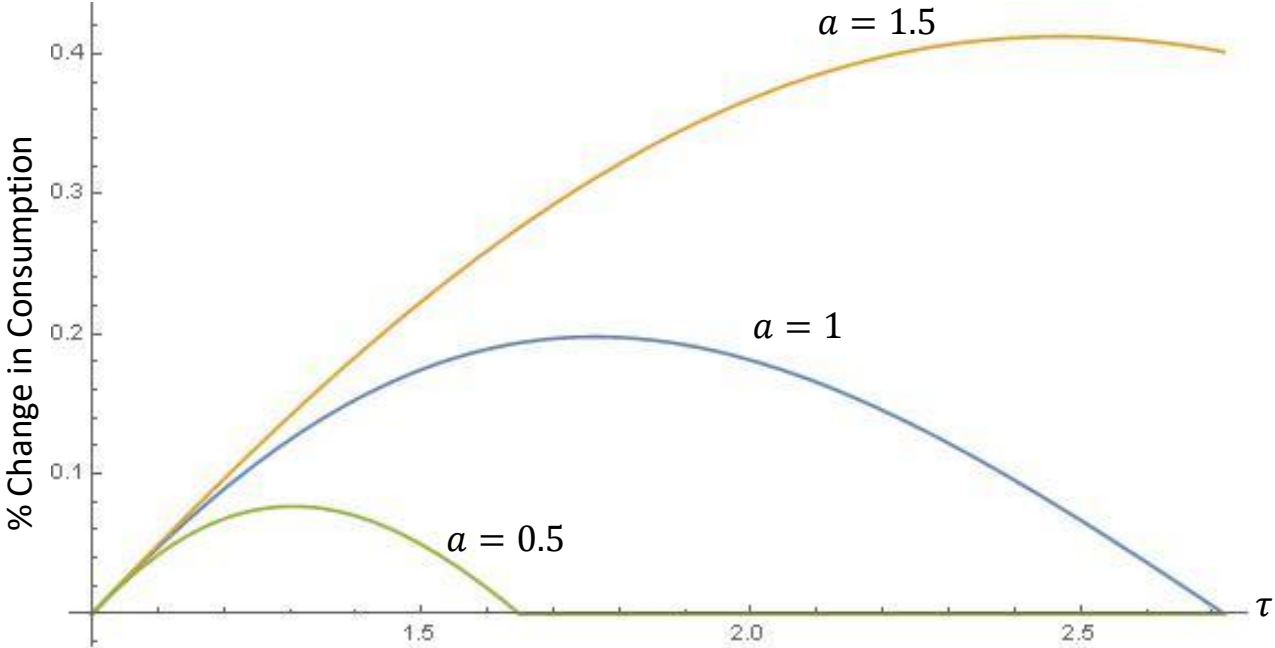
$$c_1(z) = \begin{cases} \frac{L + T_1}{e^{az}}, z \in [0, \bar{z}_2] \\ \frac{L + T_1}{\tau e^{a(1-z)}}, z \in (\bar{z}_2, 1] \end{cases} ; c_2(z) = \begin{cases} \frac{L + T_2}{\tau e^{az}}, z \in [0, \bar{z}_1] \\ \frac{L + T_2}{e^{a(1-z)}}, z \in (\bar{z}_1, 1] \end{cases}$$

$$\text{Where } T_i = \frac{L \left( \frac{\tau-1}{\tau} \right)^{\bar{z}_1}}{1 - \left( \frac{\tau-1}{\tau} \right)^{\bar{z}_1}}$$



# Increased Consumption from Tariff Revenue (vs Iceberg Costs)

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# GDP and Tariffs

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Country 1's Gross Domestic Product/Real Consumption (same in symmetric model)

$$GDP_1 = \int_0^{\bar{z}_2} p_1(z)c_1(z)dz + \int_0^{\bar{z}_1} p_1^2(z)c_2(z)dz = \frac{L}{1 - \left(\frac{\tau - 1}{\tau}\right)\bar{z}_1}$$

- Can see how GDP and real GDP change as tariffs change
- Increasing in productivity level ( $a$ ) and country size ( $L$ ), decreasing in tariffs ( $\tau$ )

# Real GDP and Tariffs: Base Prices

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Real GDP is value of production evaluated at base period prices

- Let the base period be at  $\tau = 0$ , then base prices are

$$\tilde{p}(z) = \begin{cases} e^{az}, & z \in \left[0, \frac{1}{2}\right] \\ e^{a(1-z)}, & z \in \left(\frac{1}{2}, 1\right] \end{cases}$$

# Real GDP and Tariffs

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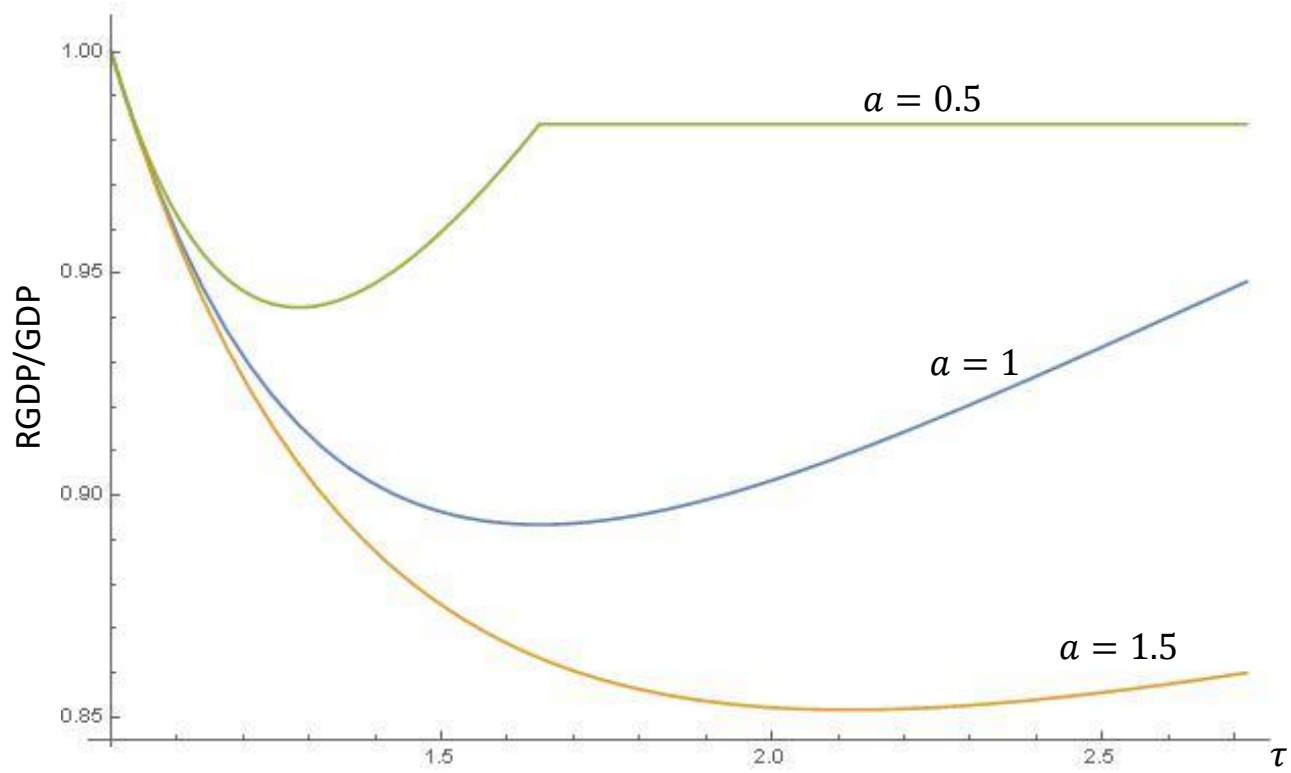
Country 1's Real Gross Domestic Product (constant price GDP)

$$\begin{aligned}RGDP_1 &= \int_0^{\bar{z}_2} \tilde{p}(z)c_1(z)dz + \int_0^{\bar{z}_1} \tilde{p}(z)c_2(z)dz \\ &= GDP_1 \left( \frac{a + \tau - 1 + a\tau - \log \tau}{2a\tau} \right)\end{aligned}$$

Note that  $RGDP_1 = GDP_1$  if  $\tau = 1$ , otherwise  $RGDP_1 \leq GDP_1$

# Tariffs: Current vs Constant Price GDP

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# Overview of DFS

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Dornbusch, Fischer, and Samuelson (1979):

- Ricardian model: 2 countries, 1 factor of production, continuum of goods

Strengths:

- Simple and intuitive. Can be used to think about effects of trade policy and trade costs.

Weaknesses:

- No explanation for why countries differ in productivity for producing goods
- Not straightforward to extend model to multiple countries

# Application of DFS: Tariff Wars

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Opp (2010):

- Similar model setup: 2 countries, continuum of goods, labor only factor of production
- Constant Elasticity of Substitution (CES) preferences

$$U = \left( \int_0^1 \theta(z)^{\frac{1}{\sigma}} (c(z))^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}}$$

Note:  $\sigma = 1 \Rightarrow$  Cobb-Douglas

Question: What is the optimal tariff rate schedule?

# Tariff Wars and the Prisoner's Dilemma

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Holding fixed partner's tariff: optimal tariff is non-zero

- Partner may retaliate and charge non-zero tariffs in return
- Global welfare always maximized by free trade

Two subquestions:

1. Without commitment to free trade, what is the Nash equilibrium for tariff rates?
2. When, if ever, will one of the countries prefer the above to free trade?



# Optimal Tariffs: Propositions

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1. Optimal tariffs schedule is uniform across goods
2. Lerner (1936) Symmetry: Import tariff is equivalent to export tax
3. Unique interior NE in tariff rates that dominates any no-trade NE
4. NE tariff rates are increasing in degree of comparative advantage and decreasing in transportation costs

# Optimal Tariffs: Main Result and Intuition

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If a country is sufficiently large (productivity adjusted labor), it will prefer NE tariffs to free trade

Tradeoffs of having tariffs:

+Intensive Margin: Gain from terms-of-trade effects (relative price of exports to imports)

-Extensive margin: Lose from having to produce goods that could have been imported

If country is large enough, terms-of-trade effects dominate

- Implications for self-regulating free trade agreements