Exercise:

Exercise 1: Newton's Method

We are going to use Newton's Method to find the root of a function. The function we will find the root of is $2x^2 - 8$.

a. Write a function, $f(x)$, that takes as an input x and returns $2x^2 - 8$
b. Write another function, $f'(x)$, that takes as an input x and returns $4x$ (this is the derivative of the above function).
c. Now, write as a function that takes x as an input and uses the above two functions to compute and returns $y = x - \frac{f(x)}{f'(x)}$
d. Start by assigning `x` any value, and define a variable `diff = 1`. Write a while loop that stores your value of `x` in a variable called `xold`, then plugs in your value of `x` into the function from (c) and replaces it with the result from the function. Each loop, compute the difference (absolute value using the `abs()` function) between the new `x` and `xold` and assign this difference to `diff`. Continue the While loop until this difference is less than 1/1000.

Exercise 2: Vectors, Logical Statements, and Loops

a. Initialize an empty vector, `vec`. Then fill the vector with twenty five each of 1's, 2's, and 3's using an if else statement embedded in a for loop (first 25 elements should be 1, second 25 elements should be 2, etc). The loop should be `i` from 1 to 75, where the loop is the index for the element. The if-else statement can use this index to decide what value to assign (the conditional statements in the if statement should be less than or equal to functions).
b. After the vector is created, create a new vector that replaces all the 2's with 9's using the old vector's name, square brackets, and a logical statement.
c. Use a for loop to sum the values of the vector created after step b; do this by initializing a variable called `sum_vec` to zero, and then each loop add an element of the vector to `sum_vec`.
d. Verify your answer using the `sum()` function.

Exercise 3: Consumer's Problem

Suppose a consumer has income equal to `Inc`, and has Cobb-Douglas preferences with $\theta_1 = \theta$ and $\theta_2 = (1 - \theta)$, so the solution to the consumer's problem is

$$c_m = \theta_m \frac{\text{Income}}{p_m}$$

a. Write a function that takes three inputs. The first input is the vector of prices (p_1, p_2), the second input is θ, and the third input is income. Then have the function return the vector of consumptions (c_1, c_2).
b. For this consumer's problem only, what are the exogenous and endogenous variables? Think carefully, since it's not the same as for the equilibrium as a whole.
c. Suppose that good 2 is imported from a foreign country, and that there is a tariff on imports equal to τ. Suppose base income is 10, plus whatever the tariff revenue is. Then we have that Income $= 10 + (\tau - 1)p_2c_2$. We can plug that into our consumer problem to see that

$$c_2 = (1-\theta) * \left(\frac{10 + (\tau - 1)p_2 c_2}{p_2}\right)$$

Which can be solved analytically to get

$$c_2 = (1-\theta)\frac{10}{p_2(1-\theta+\tau\theta)}$$

Write a function that takes as an input τ and p_2 and returns tariff revenue using the above

$$\text{Tariff Revenue} = (\tau - 1) * p_2 c_2$$

d. Write a function that takes two inputs, the first is τ, and the second is the name of a function with three inputs. The new function takes the old function and gives it the first input `c(1,τ)` [p_2 is equal to $\tau * 1$, p_1 is equal to 1], and the second input is `0.5` [$\theta = 0.5$]; and the third input is $10 + \text{tariff revenue}$. The new function returns the value of the old function with those inputs as well as tariff revenue equal to $(\tau - 1) * c_2$.

e. Use your function from part c with the input $\tau = 1.5$. Verify that constant expenditure shares hold. Note that expenditure on good 2 is $\tau p_2 c_2$ and income is 10+tariff revenue.

f. Create a new function which takes as the first input the vector of consumption (c_1,c_2); takes as a second input θ and computes a measure of utility

$$U = (c_1)^\theta (c_2)^{1-\theta}$$

What is the value of utility from part d?

Note: You can skip steps c–f and use the code here: http://www.r-fiddle.org/#/fiddle?id=CnfHTkMe

g. It is often useful to generate random numbers in R. You can generate uniform random numbers using the `runif()` function. `runif(length)` creates a vector of length `length` where each element is a random number between zero and 1. Create a length 100 vector of random numbers between 0.5 and 1.5 using this function.

h. Create a for loop that loops over each input of the vector from part f, using the element value as the input τ in c and computes utility of the consumption allocation using the input from part c. Store the value of utility for each τ in a vector named `util`.

i. Using the `plot()` function, plot τ versus `util`. Use `help("plot")` to see how to use the plot function. Make sure τ is on the horizontal axis, where does utility seem to be maximized?