ECO 745: Theory of International Economics — Jack Rossbach

Fall 2015: Problem Set 1

Problem 1. A Ricardian Framework with Transfers and Non-traded Goods

Consider the following economy based off of Dornbusch, Fisher, and Samuelson (1977). There are two countries, $i, j = 1,2$, and consumers in each country have Cobb-Douglas preferences over the set of goods $z \in [0,1]$:

$$U_i = \int_0^1 \log c_i(z)\, dz.$$

The total supply of labor in country i is L_i and goods are produced according to the production technology

$$y_i(z) = \frac{l_i(z)}{a_i(z)},$$

where

$$a_1(z) = e^{az}$$

$$a_2(z) = e^{a(1-z)}$$

Questions

1.i) Suppose that there is free trade between the two countries, but that Country 1 is forced to pay a flat transfer T to Country 2**.** Define an equilibrium for this economy.

1.ii) Show how relative wages are affected by T in this economy

1.iii) Suppose there is a now non-traded good in each country, $c_{i,0}$, so that preferences are

$$U_i = \log c_{i,0} + \int_0^1 \log c_i(z)\, dz$$

and the production technology for the non-traded good is $y_{i,0} = l_{i,0}$.

Repeat exercises 1.i) and 1.ii) for this new economy.

Problem 2. Learning-by-doing and Scale Effects in a Ricardian Framework

Consider the following dynamic economy based off of Alwyn Young (1991). There are two countries, $i, j = 1,2$, and consumers in each country have preferences over the set of goods $z \in [0, \infty)$ given by

$$U_i = \int_0^\infty e^{-\rho t} \left[\int_0^\infty \log(c_i(z,t) + 1) dv \right] dt,$$

where t is time and ρ is the discount rate. The total supply of labor in country i is L_i and goods are produced according to the production technology

$$y_i(z,t) = \frac{l_i(z,t)}{a_i(z,t)}.$$

In both countries we have the following learning-by-doing equations:

$$a_i(z,t) \geq \bar{a}(z) = e^{-z}, \qquad \forall z \in [0,\infty) \qquad (2.1)$$

$$\frac{\dot{a}_i(z,t)}{a_i(z,t)} = \begin{cases} -\int_0^\infty b_i(z',t) l_i(z',t) dz' , & if\ a_i(z,t) > \bar{a}(z) \\ 0, & if\ a_i(z,t) > \bar{a}(z) \end{cases}, \qquad (2.2)$$

$$b_i(z',t) = \begin{cases} b, & if\ a_i(z',t) > \bar{a}(z') \\ 0, & if\ a_i(z',t) = \bar{a}(z') \end{cases}, \qquad (2.3)$$

where $b > 0$. In country i at time $t = 0$, let there be a good $T_i(0)$ such that

$$a_i(T_i(0),0) = \begin{cases} \bar{a}(z), & if\ z \leq T_i(0) \\ e^{z-2T_i(0)}, & if\ z > T_i(0) \end{cases}.$$

Questions

2.i) Describe each of the learning by doing equations (2.1–2.3) in words

2.ii) Define a equilibrium featuring free trade between the two countries

2.iii) This economy features scale effects, describe what that means and why it occurs in this model.

2.iv) Suppose Country 2 is much more advanced than Country 1 ($T_2(0) > T_1(0)$). Under free trade, how much larger must Country 1 be to grow faster than and eventually overtake Country 2?

Problem 3. The Main Theorems in the 2x2x2 Heckscher-Ohlin Framework

Consider the following economy based off the standard 2x2x2 Heckscher-Ohlin model. There are two countries, $i,j = 1,2$, and consumers have Cobb-Douglas preferences over the two goods $z = 1,2$:

$$U_i = \theta_1 \log c_{i,1} + \theta_2 \log c_{i,2},$$

where $\theta_z > 0, z = 1,2$ and $\theta_1 + \theta_2 = 1$. Consumers in country i are endowed with capital, K_i, and labor, L_i, such that

$$\frac{K_1}{L_1} > \frac{K_2}{L_2}.$$

Goods z is produced using both capital and labor with the same production technology in each country:

$$y_{i,z} = A_z (k_{i,z})^{\alpha_z} (l_{i,z})^{1-\alpha_z}$$

where $1 > \alpha_1 > \alpha_2 > 0$.

Questions

3.i) Define and derive the autarky equilibrium for this economy.

3.ii) Now suppose that the two countries are able to engage in free trade. Define the equilibrium for this economy under free trade.

3.iii) Derive the conditions under which both goods are produced by both countries.

3.iv) Prove the Stolper-Samuelson, Rybcznski, and Heckscher-Ohlin theorem for this economy.

Problem 4. Revealed Comparative Advantage

Consider the following index from Balassa (1965)

$$RCA_i(z) = \left(\frac{X_i(z)}{X_i(total)}\right) / \left(\frac{X_{world}(z)}{X_{world}(total)}\right)$$

Where $X_i(z)$ is country i's exports of good z, $X_i(total)$ is country i's total exports, $X_{world}(z)$ is the World's exports of good z, and $X_{world}(total)$ is the World's total exports.

If $RCA_i(z) > 1$, we say that country i has a revealed comparative advantage in good z.

Questions

4.i) Choose a country and two years at least 5 years apart. Compute the RCA Index for each product that the country exports (either 4 digit SITC or 6 digit HS) at the earlier date.

4.ii) Compute the growth in exports between the two dates at both the aggregate level as a fraction of the country's GDP and at the product level.

4.iii) Run a simple regression to determine whether products with a revealed comparative advantage experienced more growth on average and report your results.

4.iv) Repeat the previous exercise, this time using the set of products with the strongest revealed comparative advantage (choose a RCA Index cutoff, X, such that only 10 percent of products with a RCA Index > 1 also satisfy RCA Index $> X$). Do your results change or are they the same?

Email me your data/code after turning in your problem set and keep it for use in later assignments.