ECO 745: Theory of International Economics Jack Rossbach

Fall 2015: Problem Set 2

Problem 1. Properties of CES (Dixit-Stiglitz) Preferences

Consider an economy in which there are M goods, and consumers have CES preferences over goods

$$U = \left(\sum_{m=1}^{M} \theta_m (c_m)^\rho \right)^{\frac{1}{\rho}}$$

where $0 < \rho < 1$, and that the consumer has the following budget constraint, where I is income,

$$\sum_{m=1}^{M} p_m c_m = I$$

Questions

Note: Can renormalize values of θ_m to make derivations simpler.

1.i) Derive the CES Price index P such that $U = I/P$

1.ii) Consider the case where preferences and prices are symmetric across goods ($\theta_m = 1, p_m = p\ \forall m$). Show that these preferences exhibit love for variety, i.e. that consumer welfare is increasing in the number of goods available.

1.iii) Show that CES preferences deliver a constant elasticity of substitution across goods

1.iv) Show that we get Cobb-Douglas preferences if $\rho \to 0$.

Problem 2. CES Preferences and Markups with a Finite Number of Firms

Consider the environment in problem 1 with $\theta_m = 1, \forall m$. Assume that there are M firms that can produce differentiated output at constant marginal labor cost $y_m = z l_m$, where z is their productivity.

2.i) Assume that firms engage in Cournot competition with differentiated products. Exploit symmetry across firms to derive the expression for equilibrium markups.

2.ii) Now assume that firms instead engage in Bertrand competition with differentiated products. Exploit symmetry across firms to derive the expression for equilibrium markups.

2.iii) Now assume that there are a continuum of firms, so that instead of a sum the preferences are an integral over the mass of firms $m \in [0, M]$. Derive the expression for equilibrium markups in this case.

2.iv) Assume that firms must pay a fixed cost, f, in terms of labor to enter the market and that there is free entry. Describe how the equilibrium number of firms and the average output per firm will differ across the three economies described above (Cournot, Bertrand, and Continuum).

Problem 3. Monopolistic Competition with Heterogeneous Firms

Consider an economy in which Consumers have Cobb-Douglas preferences over a homogeneous good and a CES aggregate of differentiated goods, indexed by $m \in M$, so that consumers in country j solve

$$\max_{c_0, \{c(m)\}_{m \in M}} (1-\theta)\log c_0 + \frac{\theta}{\rho}\log\left(\int_{m \in M} \left(c(m)\right)^{\rho} dm\right)$$

Subject to their budget constraint

$$p_0 c_0 + \int_{m \in M} p(m)c(m)dm = wL + \pi$$

where consumers earn income from wages and profits redistributed from firms.

There is a fixed mass of potential entrants, μ, and each firm must pay a fixed cost, f_d, in terms of labor in order to produce output, after which firms can produce their differentiated output at constant marginal labor cost. Firms differ in their productivity so firm m can produce $z(m)$ units of output for every unit of labor employed after paying the fixed costs. Suppose that firm productivities follow a Pareto$(1, \gamma)$ distribution, $\gamma > 2$, so that the cumulative distribution function will be

$$F(z) = \begin{cases} 1 - z^{-\gamma}, & z \geq 1 \\ 0, & \text{otherwise} \end{cases}$$

And the probability density function will be $dF(z) = \gamma z^{-\gamma-1}$ for $z \geq 1$.

3.i) Define an equilibrium for this economy.

3.ii) Suppose that there is a cutoff productivity $\bar{z}$, such that firms will earn non-negative profits iff $z(m) \geq \bar{z}$. Solve for $\bar{z}$ in terms of parameters of the model. Solve for the mass of firms that will produce in equilibrium, M, in terms of $\bar{z}$ and the mass of potential entrants, μ. Point out why we require $\gamma > \frac{\rho}{(1-\rho)}$.

3.iii) Suppose now that there are two symmetric countries that engage in international trade. The homogeneous good can be costlessly traded across countries. Conditional on producing domestically, differentiated firms are able to pay a fixed cost, f_e, in order to export to the foreign country subject to an iceburg cost τ.

Set up the firm maximization problem and derive the optimal pricing rule for firms to charge the domestic and foreign markets.

3.iv) Define an equilibrium for this economy with international trade.

3.v) Describe the three cases in terms of equilibrium cutoff productivity levels. Suppose we are in the equilibrium case in which there are two cutoff productivities $\bar{z}_d$ and $\bar{z}_e$, such that firms only earn profits producing domestically if $z(m) \geq \bar{z}_d$ and firms only earn profits exporting if $z(m) \geq \bar{z}_e$. Solve for $\bar{z}_d$ and $\bar{z}_e$ in terms of model parameters, and solve for the equilibrium mass of firms that will produce domestically and the equilibrium mass of firms that will export in each country.

3.vi) Suppose there is costless trade between the two countries so that $\tau = 1$. Choose model parameters, $\mu > 0, \rho \in (0,1), \gamma > \frac{\rho}{1-\rho}, L > 0, \mu > 0, f_d > 0, f_e > 0, \theta \in (0,1)$, so that there are cutoff productivities for producing domestically and exporting. Compute the equilibrium of the economy for these parameters.

3.vii) Suppose that instead of costless trade, there is a positive iceburg cost to exporting so that $\tau = 1.2$. Compute the new equilibrium and describe how the mass of producers and the mass of exporters change.

If no longer in same equilibrium case with two cutoffs, explain how you know this instead of computing the new equilibrium.

Problem 4. Gravity Regressions

Download the Gravity dataset from CEPII at:

http://www.cepii.fr/CEPII/en/bdd_modele/presentation.asp?id=8

4.i) Estimate the following gravity regression for the year 2000:

$$\log X_{ij} = \log \beta_0 + \beta_1 \log Y_i + \beta_2 \log Y_j - \beta_3 \log D_{ij} + \beta_{FTA} FTA_{ij} + \epsilon_{ij}$$

Where X_{ij} are exports from country i to j (exclude observations with $X_{ij} = 0$), Y_i and Y_j are the GDPs of each country, D_{ij} is the distance between the countries, and FTA_{ij} is an indicator function equal to 1 if the countries had a free trade agreement in the year 2000.

4.ii) Re-run the gravity regression with bilateral controls for a common official language, contiguity, and a historical colonial relationship. Does the estimate on β_{FTA} change?

4.iii) Re-run the gravity equation with the bilateral controls above, but instead of using $\log Y_i$ and $\log Y_j$ use fixed effects for each country as a proxy for country size. Does the estimate on β_{FTA} change?

4.iv) Based on the estimated coefficients from the gravity regression in 4.iii), how much higher would we have expected exports to be from China to the United States in 2000 if they had a free trade agreement?

What are some issues with using the gravity regression in 4.iii) to predict the impact of a free trade agreement between China and United States?