# ECO 745: Theory of International Economics

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# Heterogeneity and Markups in Trade Models

Melitz (2003) and Chaney (2008) incorporate firm-level heterogeneity into CES trade framework

- Flexible framework for studying how trade affects reallocation among firms
- Simple pricing behavior across firms: CES with continuum $\Rightarrow$ constant markups
    - Not well suited for studying how trade affects pricing behavior/markups

Melitz and Ottaviano (2008) replace CES preferences with linear demand

- Linear (quadratic) demand system from Ottaviano, Tabushi, and Thisse (2002)
- Generates endogenous markups that depend on market environment

# Linear Demand System: Autarky Setup

Homogeneous good: $q_0^c$

Heterogeneous good with differentiated varieites: $q_i^c$, i ∈ Ω

Linear (quadratic) demand system, at the consumer level:

$$U = q_0^c + \alpha \int_{i\in\Omega} q_i^c di - \frac{1}{2}\gamma \int_{i\in\Omega} (q_i^c)^2 di - \frac{1}{2}\eta \left( \int_{i\in\Omega} q_i^c di \right)^2$$

- $\alpha > 0$, $\eta > 0$ govern substitutability between homogeneous good and heterogeneous goods
- $\gamma > 0$ governs substitutability between varieties (if $\gamma = 0$ perfect substitutes).

Budget constraint is at consumer level (normalize $p_0 = 1$) where consumers have income $I^c$:

$$q_0^c + \int_{i\in\Omega} p_i q_i^c = I^c$$

# Linear Demand System

$$\mathcal{L} = q_0^c + \alpha \int_{i\in\Omega} q_i^c di - \frac{1}{2}\gamma \int_{i\in\Omega} (q_i^c)^2 di - \frac{1}{2}\eta \left( \int_{i\in\Omega} q_i^c di \right)^2 + \lambda \left( y - q_0^c - \int_{i\in\Omega} p_i q_i^c \right)$$

First order conditions yield (let $Q^c = \int_{i\in\Omega} q_i^c di$):

$$[q_0^c]: 1 - \lambda = 0$$

$$[q_i^c]: \; \alpha - \gamma q_i^c - \eta Q^c - \lambda p_i = 0$$

Which, for $q_0^c > 0$, yields

$$p_i = \alpha - \gamma q_i^c - \eta Q^c$$

Note that marginal utilities are bounded, so may not have positive demand for a good

# Solving for Demand

Can invert previous equation for $q_i^C$

$$q_i^c = \frac{1}{\gamma}(\alpha - p_i - \eta Q^c)$$

Note if $p_i > \alpha - \eta Q^c$ then demand will be zero. This is not the case in CES framework (demand positive at all prices, although not necessarily high enough to offset fixed costs)

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Let there be mass $N$ of varieties consumed in equilibrium. Integrating over both sides of above:

$$\int_{i\in\Omega} q_i^c di = \int_{i\in\Omega} \frac{1}{\gamma}(\alpha - p_i - \eta Q^c) di$$

$$Q^c = \frac{1}{\gamma}\left(aN - \int_{i\in\Omega} p_i di - \eta N Q^c\right)$$

$$Q^C\left(1 + \frac{\eta N}{\gamma}\right) = \frac{1}{\gamma}\left(aN - \int_{i\in\Omega} p_i di\right)$$

# Solving for Consumer Demand

Define $\bar{p} = \frac{1}{N}\int_{\mathrm{i}\in\Omega} p_i$ as the average price, then substituting $Q^C$ into previous formula for $q_i^c$:

$$q_i^c = \frac{1}{\gamma}\left(\alpha - p_i - \eta\left[\frac{1}{\gamma + \eta N}\left(aN - \int_{i\in\Omega} p_i di\right)\right]\right)$$

$$q_i^c = \frac{1}{\gamma}\left(\alpha - p_i - \eta\frac{N}{\gamma + \eta N}(a - \bar{p})\right)$$

$$q_i^c = \frac{\alpha}{\eta N + \gamma} - \frac{1}{\gamma}p_i + \frac{\eta N}{\eta N + \gamma}\frac{1}{\gamma}\bar{p}$$

Note there are no income effects for differentiated varieties ($I^c$ doesn't appear above)

# Solving for Total Demand

Assume there are $L$ consumers, then $q_i \equiv Lq_i^c$, and so

$$q_i = \frac{\alpha L}{\eta N + \gamma} - \frac{L}{\gamma} p_i + \frac{\eta N}{\eta N + \gamma} \frac{L}{\gamma} \bar{p}$$

Where for a good to be consumed means that

$$p_i \leq \frac{1}{\eta N + \gamma} (\gamma \alpha + \eta N \bar{p}) \equiv p_{\max}, \qquad \forall i \in Q^c$$

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Note that $p_{\max} \leq \alpha$ and that $p_{\max}$ decreases as $N$ increases and $\bar{p}$ decreases

- Shows how $p_{\max}$ changes with competition
- More competitors ($N \uparrow$) or competitors charge lower prices ($\bar{p} \downarrow$) therefore have to charge a lower price to sell a positive quantity ($p_{\max} \downarrow$)

# Price Elasticity of Demand

Note that the price elasticity of demand is given by

$$\epsilon_i = \left|\frac{\partial q_i/q_i}{\partial p_i/p_i}\right| = \frac{1}{p_{\max}/p_i - 1} = \frac{1}{\frac{1}{\eta N + \gamma}(\gamma\alpha + \eta N\bar{p})/p_i - 1}$$

Note that the price elasticity of demand is not uniquely pinned down by substitutability across varieties, $\gamma$ (it is in CES case)

- Suppose there number of varieties increases ($N \uparrow$) or average price decreases ($\bar{p} \downarrow$), then the cutoff price falls ($p_{\max} \downarrow$) which leads to a higher price elasticity of demand ($\epsilon_i \uparrow$).

- Note also, the price elasticity of demand is increasing as a function of $p_i$

# Solving for Total Demand

Can evaluate welfare using indirect utility function (plug optimal demand into utility function):

$$U = I^c + \frac{1}{2}\left(\eta + \frac{\gamma}{N}\right)^{-1}(\alpha - \bar{p})^2 + \frac{1}{2}\left(\frac{N}{\gamma}\sigma_p^2\right)$$

where $\sigma_p^2$ represents the variance of prices

$$\sigma_p^2 = \frac{1}{N}\int_{i\in\Omega}(p_i - \bar{p})^2$$

Note that welfare increases as the average price falls ($\bar{p} \downarrow \Rightarrow U \uparrow$) and as the dispersion of prices increases $\left(\sigma_p^2 \uparrow \Rightarrow U \downarrow\right)$ as consumers shift consumption towards lower priced goods

Preferences also exhibit love for variety ($N \uparrow \Rightarrow U \uparrow$)

# Closing the Model

Need to find how firms should set prices.

Homogeneous sector produces output one-to-one with labor: $q_0 = l_0 \Rightarrow w = p_0 = 1$

Heterogeneous firms:

- Pay fixed costs $f_E$ to discover their unit cost of production: $c$
  - Distribution for $c$, $G(c)$ is common across firms, with support $\mathrm{c} \in [0, c_M]$
- After discovering $c$ firms produce with no further fixed costs
  - Note: even in absence of additional fixed costs, firms will not produce if $p_i > p_{\max}$

# Firm Pricing Problem

Firms with unit cost $c$ will solve (note $w = 1$ and $l(c) = cq(c)$)

$$\max_{q(c)} p(c)q(c) - cq(c)$$

Which, plugging in prices from consumer problem (substitute $q_i = q_i^c/L$), becomes

$$\max_{q(c)} \left(\alpha - \frac{\gamma}{L}q(c) - \eta Q^c - c\right) q(c)$$

Which has FOC:

$$[q(c)]: \alpha - 2\frac{\gamma}{L}q(c) - \eta Q^C - c \leq 0, \qquad = 0 \; if \; q(c) > 0$$

Let $c_D \equiv \alpha - \eta Q^C$ be the cutoff unit cost such that firms are indifferent between producing or not

Then $q(c) = 0$ if $c > c_D$ and if $c < c_D$ then

$$q(c) = \frac{L}{2\gamma}(c_D - c)$$

# Equilibrium Sales and Markups

Assume that $c_D < c_M$ in equilibrium, so some firms choose not to produce.

Note that $c_D$ completely characterizes equilibrium prices and markups:

$$\text{prices:} \quad p(c) = \frac{1}{2}(c_D + c)$$

$$\text{output:} \quad q(c) = \frac{L}{2\gamma}(c_D - c)$$

$$\text{markups:} \quad \mu(c) \coloneqq p(c) - c = \frac{1}{2}(c_D - c)$$

$$\text{sales:} \quad r(c) \coloneqq p(c)q(c) = \frac{L}{4\gamma}(c_D^2 - c^2)$$

The cutoff also characterizes production profits

$$\text{production profits:} \quad \pi(c) \coloneqq p(c)q(c) - cq(c) = \frac{L}{4\gamma}(c_D - c)^2$$

# Free Entry

Firms enter until expected profits are zero, i.e. expected production profits equal cost of productivity draw $(f_e)$. Plugging in the expression for profits determines $c_D$ (given $G(c)$):

$$\int_0^{c_D} \pi(c) dG(c) = \frac{L}{4\gamma} \int_0^{c_D} (c_D - c)^2 dG(c) = f_E$$

Note also that by the definition of the cutoff unit cost, we must have $p(c_D) = p_{\max}$, therefore

$$p(c_D) = \frac{1}{2}(c_D + c_D) = \frac{1}{\eta N + \gamma}(\gamma\alpha + \eta N \bar{p}) = p_{\max}$$

Which we can rearrange to find the number of firms that produce in equilibrium

$$N = \frac{2\gamma}{\eta} \frac{\alpha - c_D}{c_D - \bar{c}}$$

Where $\bar{c} = \int_0^{c_D} c \; dG(C) / G(c_D)$ is the average unit cost of producing firms.

Note, the number of entrants is: $N_E = N/G(c_D)$

# Pareto Productivity Distribution

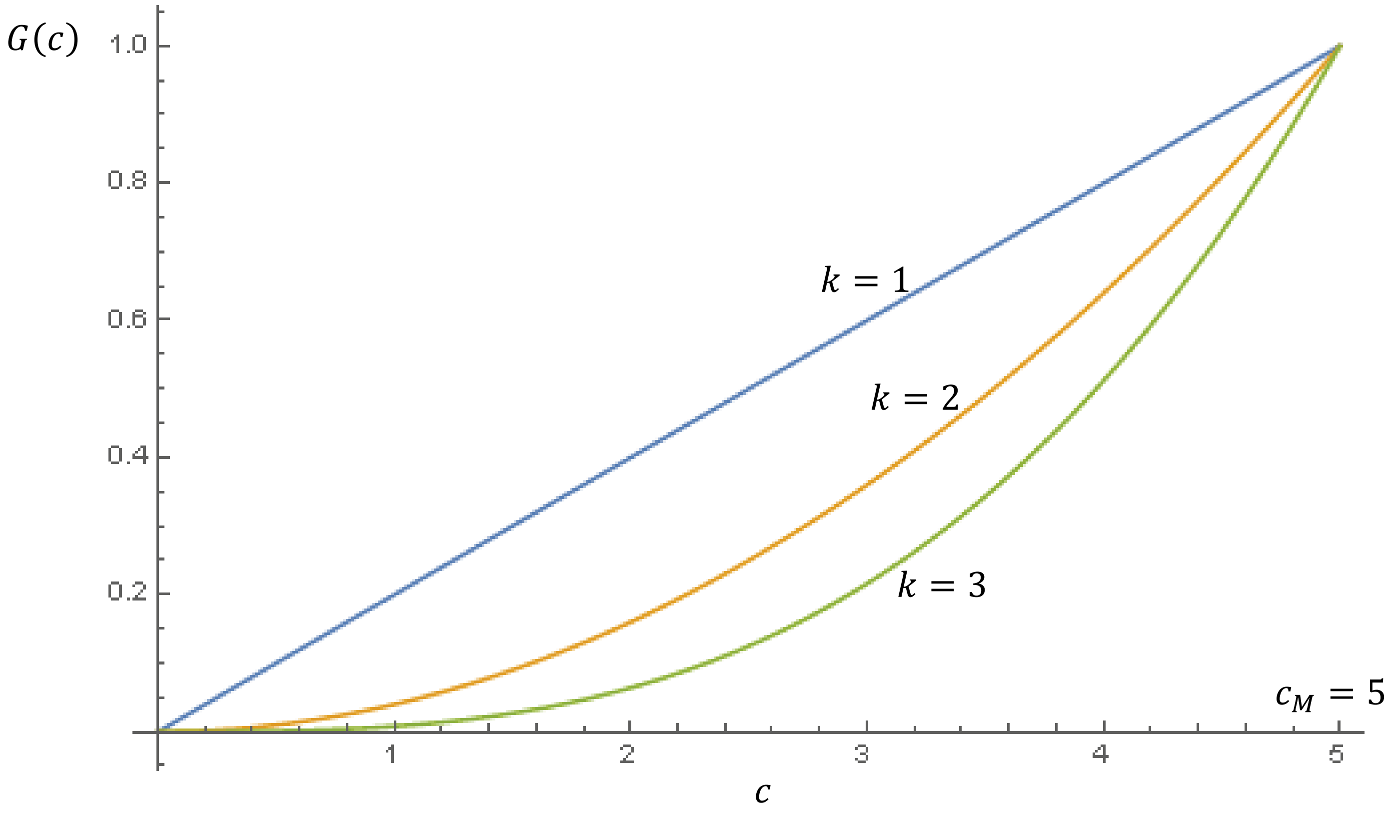
To completely characterize equilibrium, need unit cost distribution, $G(c)$

Assume that productivity $(1/c)$ follows a Pareto($1/c_M, k$) distribution, then

$$G(c) = \left(\frac{c}{c_M}\right)^k, \qquad c \in [0, c_M]$$

where $k \geq 1$. Note $k$ indexes the dispersion of cost draws, if $k = 1$ then $G(c)$ is a uniform distribution, and as $k$ increases the proportion of high unit cost draws increases.

# Unit Cost Distribution Function



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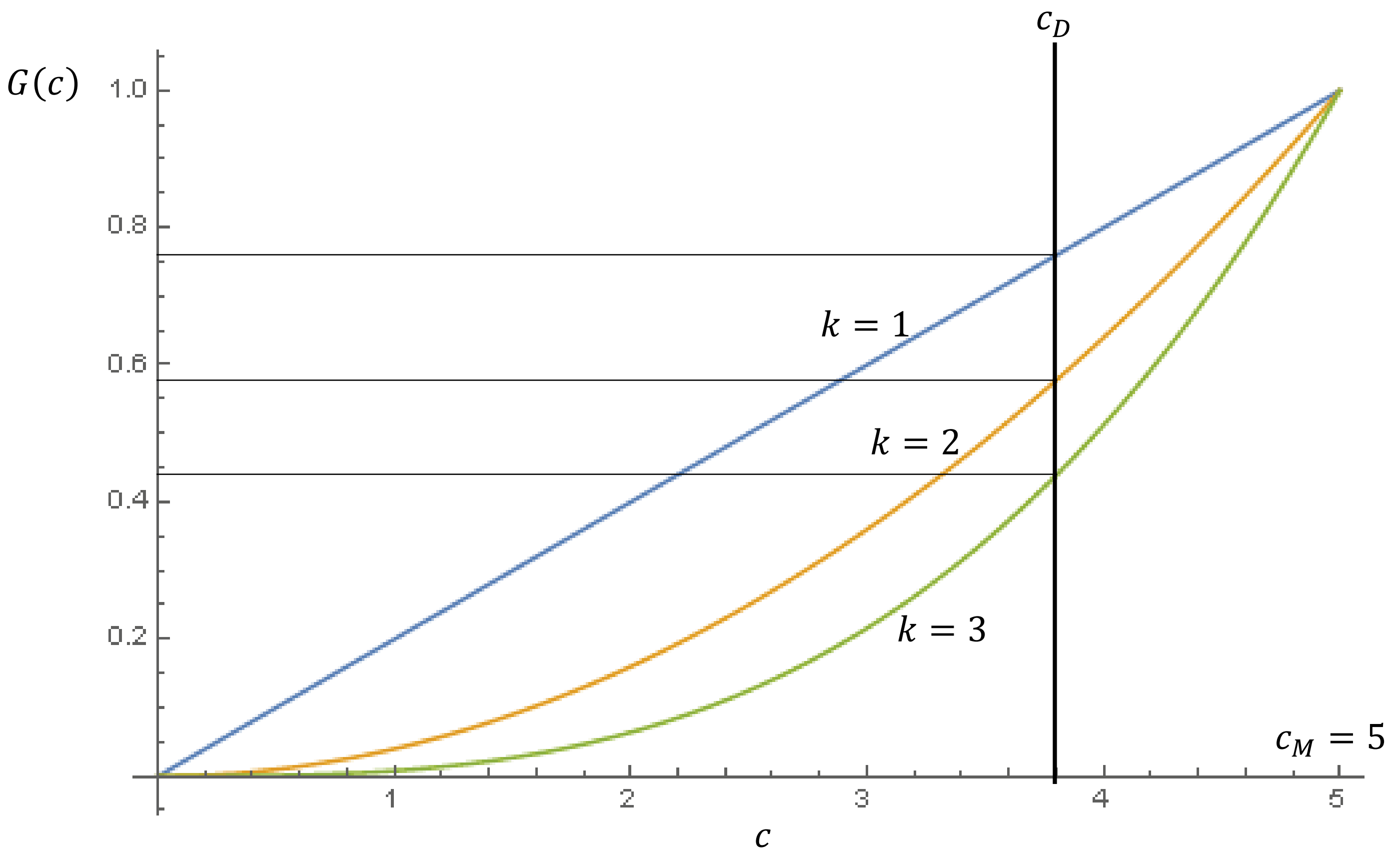
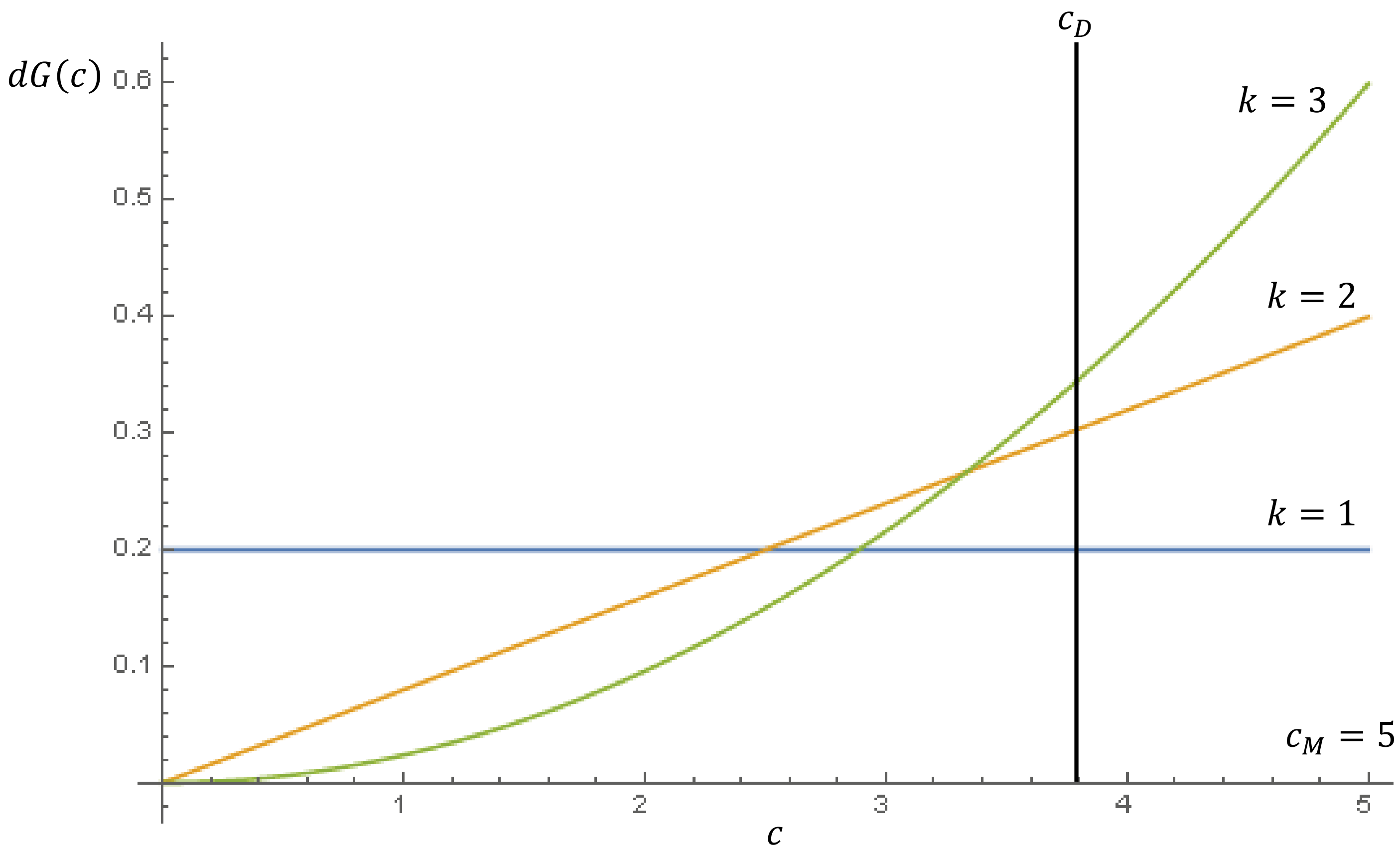


# Unit Cost Density Function



# Free Entry with Pareto Distribution

Note that $dG(c) = (k/c_M)(c/c_M)^{k-1}$, therefore free entry condition becomes

$$\frac{L}{4\gamma}\int_0^{c_D}(c_D - c)^2\left(\frac{k}{c_M}\right)\left(\frac{c}{c_M}\right)^{k-1} dc = f_E$$

Which, after integrating and solving for $c_D$, yields

$$c_D = \left(\frac{2(k+1)(k+2)\gamma(c_M)^k f_E}{L}\right)^{\frac{1}{k+2}}$$

Note that the cutoff and average unit cost of producers are lower when markets are larger ($L \uparrow \Rightarrow c_D \downarrow$), when entry costs are lower ($f_E \downarrow \Rightarrow c_D \downarrow$), and when differentiated varieties are more substitutable ($\gamma \downarrow \Rightarrow c_D \downarrow$)

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Note also that the variance of unit costs of producers decreases with market size

$$\int_0^{c_D}(c_D - c)^2 dG(c) = \frac{4\gamma}{L} f_E$$

# Average Equilibrium Sales and Markups

We can exploit the Pareto distribution to compute averages of firm-level variables in terms of $c_D$

$$\text{average unit cost:} \quad \bar{c} = \frac{k}{k+1} c_D$$

$$\text{average price:} \quad \bar{p} = \frac{2k+1}{2k+2} c_D$$

$$\text{average markups:} \quad \bar{\mu} = \frac{1}{2} \frac{1}{k+1} c_D$$

$$\text{average output:} \quad \bar{q} = \frac{L}{2\gamma} \frac{1}{k+1} c_D = \frac{(k+2)(c_M)^k}{(c_D)^{k+1}} f_E$$

$$\text{average sales:} \quad \bar{r} = \frac{L}{4\gamma} \frac{1}{k+2} c_D^2 = \frac{(k+1)(c_M)^k}{(c_D)^k} f_E$$

$$\text{average production profits:} \quad \bar{\pi} = f_E (c_M / c_D)^k$$

Note that the above are unweighted averages (i.e. average unit cost isn't weighted by firm output)

# Welfare with Pareto Distribution

The unit cost cutoff completely characterizes the equilibrium and welfare can be written as

$$U = 1 + \frac{1}{2\eta}(\alpha - c_D)\left(\alpha - \frac{k+1}{k+2}c_D\right)$$

Which tells us that welfare is higher if the unit cost cutoff is lower

Note that falls in $c_D$ lead to welfare gains from both higher average productivity and from lower markups (pro-competitive welfare gains)

# Frictionless Integration

Suppose we have two countries, $H$ and $F$, that are able to engage in costless free trade

- Market size becomes $L = L^H + L^F$. Cutoffs in the home country fall from $c_D^H$ to $c_D$ where

$$\frac{c_D}{c_D^H} = \frac{\left(\frac{2(k+1)(k+2)\gamma(c_M)^k f_E}{L}\right)^{\frac{1}{k+2}}}{\left(\frac{2(k+1)(k+2)\gamma(c_M)^k f_E}{L^H}\right)^{\frac{1}{k+2}}} = \left(\frac{L^H}{L^H + L^F}\right)^{\frac{1}{k+2}}$$

Therefore average prices and markups go down, average output, sales, and production profits increase, and welfare increases as well

- Results for markups are interesting: bigger firms charge higher markups, and firms are bigger in integrated economy, but markups are lower due to increased competition

- Note that the variance of prices, markups, and unit costs of producers increases while variance of output and sales decreases in integrated equilibrium

# Trade with Iceberg Costs

Now suppose there is an iceburg cost $\tau^l$ to export to country $l = H, F$

Countries defined by their market size $L^l$ and barriers to imports $\tau^l$

- Note still have same formula for cutoff price:

$$p_{\max}^l = \frac{1}{\eta N^l + \gamma}\left(\gamma\alpha + \eta N^l \bar{p}^l\right)$$

- Where $N^l$ is the number of firms that serve market $l$ and $\bar{p}^l$ is the average price charged in $l$

# Firm Problem under Iceberg Costs

Let $p_D^l(c), q_D^l(c)$ be the domestic price and output and $p_X^l(c), q_X^l(c)$ be the price and output of exports for a firm with productivity $c$ headquartered in market $l$

Due to linearity of production function, can again consider optimization separately for each market after paying fixed cost $f_E$ to draw unit output cost from $G(c)$

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Due to linearity of production function, can again consider optimization separately for each market after paying fixed cost $f_E$ to draw unit output cost from $G(c)$

Firms maximize domestic production profits

$$\max_{q_D^l(c)} p_D^l(c) q_D^l(c) - c q_D^l(c) = \left(\alpha - \frac{\gamma}{L^l} q_D^l(c) - \eta Q^l - c\right) q_D^l(c)$$

As well as profits from exporting, for $h \neq l$

$$\max_{q_D^l(c)} p_X^l(c) q_X^l(c) - \tau^h c q_X^l(c) = \left(\alpha - \frac{\gamma}{L^h} q_X^l(c) - \eta Q^h - c\right) q_X^l(c)$$

Note that from FOC we have: $q_D^l(c) = \left(L^l/\gamma\right)\left[p_D^l(c) - c\right]$ and $q_X^l = \left(L^l/\gamma\right)\left[p_X^l(c) - \tau c\right]$

# Firm Maximization

$$\max_{q_D^l(c)} p_D^l(c) q_D^l(c) - c q_D^l(c) = \left(\alpha - \frac{\gamma}{L^l} q_D^l(c) - \eta Q^l - c\right) q_D^l(c)$$

FOC for domestic problem yields

$$\left[q_D^l(c)\right]:\ \alpha - 2\frac{\gamma}{L^l} q_D^l(c) - \eta Q^l - c = 0$$

Note this implies

$$\alpha - \frac{\gamma}{L^l} q_D^l(c) - \eta Q^l - c = \frac{\gamma}{L^l} q_D^l(c)$$

$$\frac{\gamma}{L^l} q_D^l(c) = p_D^l(c) - c$$

Therefore $q_D^l(c) = \left(L^l/\gamma\right)\left[p_D^l(c) - c\right]$

Similarly, for exports we have: $q_X^l = \left(L^l/\gamma\right)\left[p_X^l(c) - \tau c\right]$

# Cutoffs, Prices, and Output

Similarly to the autarky case, cutoffs satisfy $p_D^l(c_D^l) = p_{\max}^l$ and $p_X^l(c_X^l) = p_{\max}^h/\tau^h$

For producers in $l$ with $c > c_D^l$ we have

$$p_D^l(c) = \frac{1}{2}(c_D^l + c), \qquad q_D^l(c) = \frac{L^l}{2\gamma}(c_D^l - c)$$

Which mirrors the autarky expression, except $c_D^l \neq c_D$. For producers in $l$ with $c > c_X^l$ we have:

$$p_X^l(c) = \frac{\tau^h}{2}(c_X^l + c), \qquad q_X^l(c) = \frac{L^h}{2\gamma}\tau^h(c_X^l - c)$$

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$$p_X^l(c) = \frac{\tau^h}{2}(c_X^l + c), \qquad q_X^l(c) = \frac{L^h}{2\gamma}\tau^h(c_X^l - c)$$

Using the expressions above we have profits from domestic production and exporting:

$$\pi_D^l(c) = \frac{L^l}{4\gamma}(c_D^l - c)^2, \qquad \pi_X^l(c) = \frac{L^h}{4\gamma}(\tau^h)^2(c_X^l - c)^2$$

# Free Entry with Iceberg Costs

Firms will enter until expected production + exporting profits are equal to entry costs

$$\int_0^{c_D^l} \pi_D^l(c)dG(c) + \int_0^{c_X^l} \pi_X^l(c)dG(c) = f_E$$

Again assume a Pareto distribution for productivity in both countries, therefore above becomes:

$$\int_0^{c_D^l} \left[\frac{L^l}{4\gamma}\left(c_D^l - c\right)^2\right]\left(\frac{k}{c_M}\right)\left(\frac{c}{c_M}\right)^{k-1} dc + \int_0^{c_X^l} \left[\frac{L^h}{4\gamma}\left(\tau^h\right)^2\left(c_X^l - c\right)^2\right]\left(\frac{k}{c_M}\right)\left(\frac{c}{c_M}\right)^{k-1} dc = f_E$$

Which, after integrating and simplifying, yields

$$L^l\left(c_D^l\right)^{k+2} + L^h\left(\tau^h\right)^2\left(c_X^l\right)^{k+2} = \gamma\phi$$

Where $\phi = 2(k+1)(k+2)(c_M)^k f_E$ is a technology index that depends only the productivity distribution and entry costs (note above expression is same as in autarky)

# Cutoffs under Iceberg Costs

Assume we're in the equilibrium with $c_D^l < c_M$, then cutoffs are determined by

$$L^l\left(c_D^l\right)^{k+2} + L^h\left(\tau^h\right)^2\left(c_X^l\right)^{k+2} = \gamma\phi$$

Along with the relative cutoffs for earning zero profits domestically vs by exporting:

$$c_X^h = \frac{c_D^l}{\tau^l}$$

Therefore we can characterize cutoffs by the system:

$$L^l\left(c_D^l\right)^{k+2} + L^h\rho^h\left(c_D^h\right)^{k+2} = \gamma\phi, \qquad l = H, F; h \neq l$$

Where $\rho^h = \left(\tau^h\right)^{-k} \in (0,1)$ is an inverse measure of trade costs

# Cutoffs under Iceberg Costs

Solving the previous system for the cutoffs yields

$$c_D^l = \left[\frac{\gamma\phi}{L^l}\frac{1-\rho^h}{1-\rho^l\rho^h}\right]^{\frac{1}{k+2}}, \qquad l = H, F$$

$$c_X^l = \frac{1}{\tau^h}\left[\frac{\gamma\phi}{L^h}\frac{1-\rho^l}{1-\rho^h\rho^l}\right]^{\frac{1}{k+2}}, \qquad l = H, F$$

Similarly we can find the number of firms selling in country $l$ as

$$N^l = \frac{2(k+1)\gamma}{\eta}\frac{\alpha - c_D^l}{c_D^l}$$

# Trade Implications

Under trade, we have the following model implications

- Only the most productive firms export $\left(c_X^l < c_D^l\right)$
- Fewer domestic firms than in autarky $\left(c_D^l < c_D\right)$
- Exporters charge lower markups in foreign market

# Welfare Implications

Welfare can again be expressed as

$$U^l = 1 + \frac{1}{2\eta}\left(\alpha - c_D^l\right)\left(\alpha - \frac{k+1}{k+2}c_D^l\right)$$

Three welfare channels from opening to trade or a fall in iceberg costs:

- Increased set of varieties available to consumers
- Reallocation from less productive to more productive firms
- Decreased markups (pro-competitive gains, absent from Melitz(2003))

# Unilateral Liberalization

Suppose home country lowers import barriers while foreign country doesn't

- No change in unit cost cutoffs for selling to foreign market
- Number of entrants in home country decreases, number of entrants in foreign country increases
- Home country loses more domestic entrants than it gains foreign exporters
  - Therefore level of competition rises in the foreign country, and falls in the home country
  - Welfare decreases in home country, increases in foreign country (bad to unilaterally liberalize)
  - Intuition is by liberalizing, home country is less attractive to entering firms. Firms in foreign country have same access to home country market, but are protected from home firms in domestic foreign country market