ECO 745: Theory of International Economics

Jack Rossbach

Fall 2015 - Lecture 12

Basic Ricardian Model

Dornbusch, Fisher, Samuelson (1979)

- Ricardian model with 2 countries, 1 factor of production, and a continuum of goods
- Countries differ in relative productivities for producing different goods
- Open to trade ⇒ countries to specialize in the goods they have comparative advantages in

Difficulty: How to extend to a multi-country framework?

Comparative Advantage in Ricardian Models

Let country i produce good k with unit labor cost a_k^i, i.e. $y = l_{i,k,}/a_k^i$

With N goods and 2 countries, can order goods by relative comparative advantage

$$a_1^1/a_1^2 < a_2^1/a_2^2 < \cdots < a_N^1/a_N^2$$

Similarly with 2 goods and J countries, can order countries by relative comparative advantage

$$a_1^1/a_2^1 < a_1^2/a_2^2 < \cdots < a_1^J/a_2^J$$

In both cases, there will be a cutoff in the chains that determines patterns of production

- In first chain, country 1 will specialize in goods to left of cutoff, country 2 in goods to right
- In second chain, countries to left of cutoff will produce good 1, countries to right good 2

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Comparative Advantage in Multi-dimensional Ricardian Models

Difficulty: How to construct such chains with N goods and J countries? Potential Solutions:

- Jones (1961): Look at environments in which each country produces only one good
- Wilson (1980): No need to predict pattern of trade, can determine comparative statics such as changes in prices and welfare without it.
- Costinot (2009): Restrict attention to environments in which unit labor costs are log-submodular in good characteristics and country characteristics, so comparative advantage chains can be constructed
- Eaton-Kortum (2002): Assume productivities are drawn from a Frechet distribution. Like Wilson, doesn't predict pattern of trade, but comparative statics much simpler and better suited for empirical work.

Brief Intuition of Costinot (2009)

- N goods, with good characteristics $\sigma_k \in \Sigma$
- J countries, with country characteristics $\gamma_j \in \Gamma$
- $a(\sigma, \gamma)$ = unit labor requirement for good with characteristics σ in country with characteristics γ

Log-Submodularity: $a(\sigma, \gamma)$ is strictly log-submodular if for any $\sigma > \sigma'$ and $\gamma > \gamma'$:

$$a(\sigma, \gamma)a(\sigma', \gamma') < a(\sigma, \gamma')a(\sigma', \gamma)$$

Intuition: suppose σ and γ are measures of how technology intensive goods/countries are, then

- LHS: High tech country produces high tech good, low country produces low tech good
- RHS: High tech country produces low tech good, low tech country produces high tech good

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$$a(\sigma, \gamma)a(\sigma', \gamma') < a(\sigma, \gamma')a(\sigma', \gamma)$$

Results: If above is satisfied can create standard comparative advantage chains. Just rearrange:

$$\frac{a(\sigma, \gamma)}{a(\sigma', \gamma)} < \frac{a(\sigma, \gamma')}{a(\sigma', \gamma')}$$

Pattern of trade: High γ (tech) countries specialize in high σ (tech) goods

Eaton and Kortum (2002)

Basic Framework *(notational change to be consistent with paper)*

- Continuum of goods $j \in [0,1]$
- $i, n = 1,2, \ldots, N$ countries
- Country unit input costs: c_i
- Good specific productivity in country i: $z_i(j)$

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- Iceburg trade costs: $d_{ni} > 1$ for $i \neq n$ ($d_{ii} = 1$)
- Perfect competition $\Rightarrow$ price charged by firms in country i to consumers in country n for good j

$$p_{ni}(j) = \left(\frac{c_i}{z_i(j)} \right) d_{ni}$$

Consumers and Prices

Consumers have CES preferences over goods

$$U_n = \left[\int_0^1 \left(Q_n(j)\right)^{\frac{\sigma-1}{\sigma}} dj\right]^{\frac{\sigma}{\sigma-1}}$$

Total expenditures of country n are X_n

$$\int_0^1 p_n(j) Q_n(j) dj = X_n$$

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The price of good j in country n is the minimum price across producers in all countries

$$p_n(j) = \min\{p_{n1}(j), p_{n2}(j), \dots, p_{nN}(j)\}$$

$$= \min\left\{\left(\frac{c_1}{z_1(j)}\right) d_{n1}, \left(\frac{c_2}{z_2(j)}\right) d_{n2}, \dots, \left(\frac{c_N}{z_N(j)}\right) d_{nN}\right\}$$

Frechet Productivity Distribution

Assume productivity in country i follows a Frechet(T_i, θ) distribution

$$F_i(z) = P(Z \leq z) = e^{-T_i z^{-\theta}}$$

- $T_i > 0$ governs the location of the productivity distribution for country i.
  - Higher $T_i \Rightarrow$ higher productivity draw more likely for any good j
- $\theta > 0$ governs variation in the productivity distribution (common across countries)
  - Higher $\theta \Rightarrow$ less variability across goods (governs degree of comparative advantage)
  - sd$[\log z] = \pi/(\theta\sqrt{6})$; Geometric Mean $= e^{\gamma/\sigma} T_i^{1/\theta}$ ($\gamma \approx .577$, Euler's constant)

Key Property of Frechet Productivity Distribution

Why the Frechet distribution?

- The Frechet distribution is an Extreme Value (type II) distribution and is *max stable*
- Suppose $Z_1, Z_2, \ldots, Z_N$ follow Frechet(T_i, θ) distributions. Define $Z_{\max} = \max\{Z_1, Z_2, \ldots, Z_N\}$, then

$$F_{\max}(z) = e^{-\sum_{i=1}^{N} T_i z^{\theta}} = e^{-z^{\theta} \sum_{i=1}^{N} T_i}$$

So therefore $Z_{\max} \sim$Frechet$\left(\sum_{i=1}^{N} T_i \,, \theta\right)$

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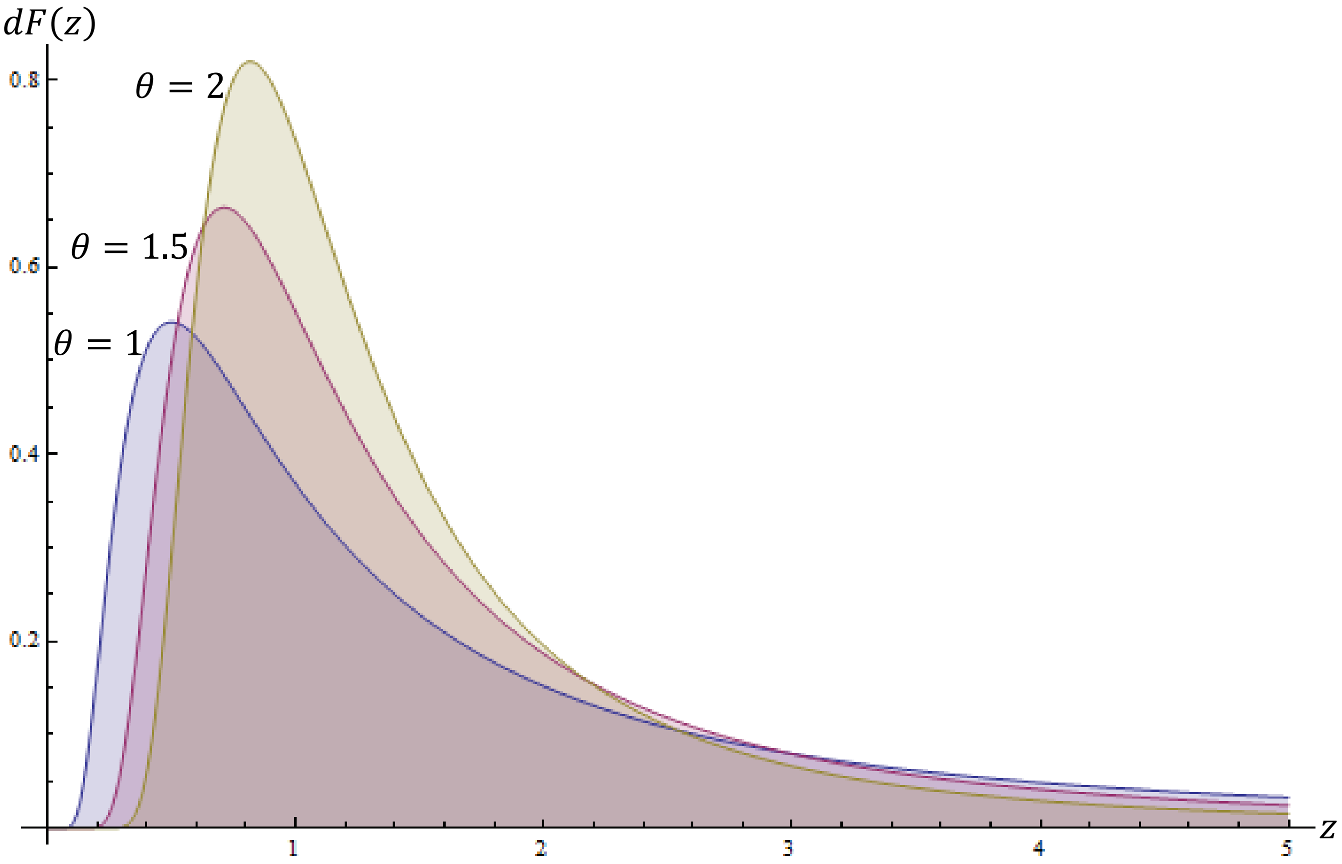
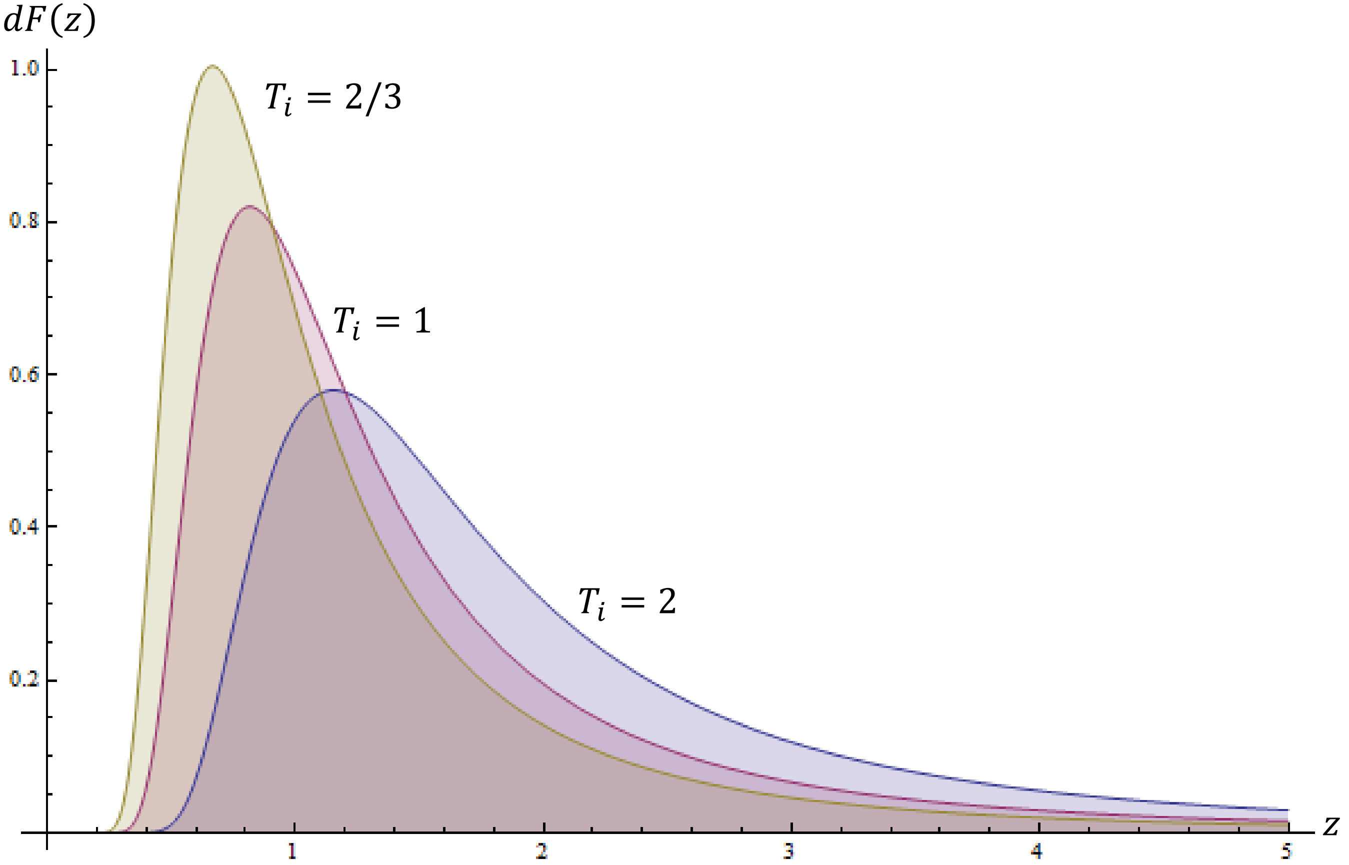
- This makes the Frechet distribution great for studying environments with perfect competition as it makes it easy to characterize the productivity of the maximal productivity producers
- This is similar to how the Pareto distribution is great for studying extensive margins, since the (left) truncated Pareto distribution is still a Pareto distribution

Density of Frechet Distribution



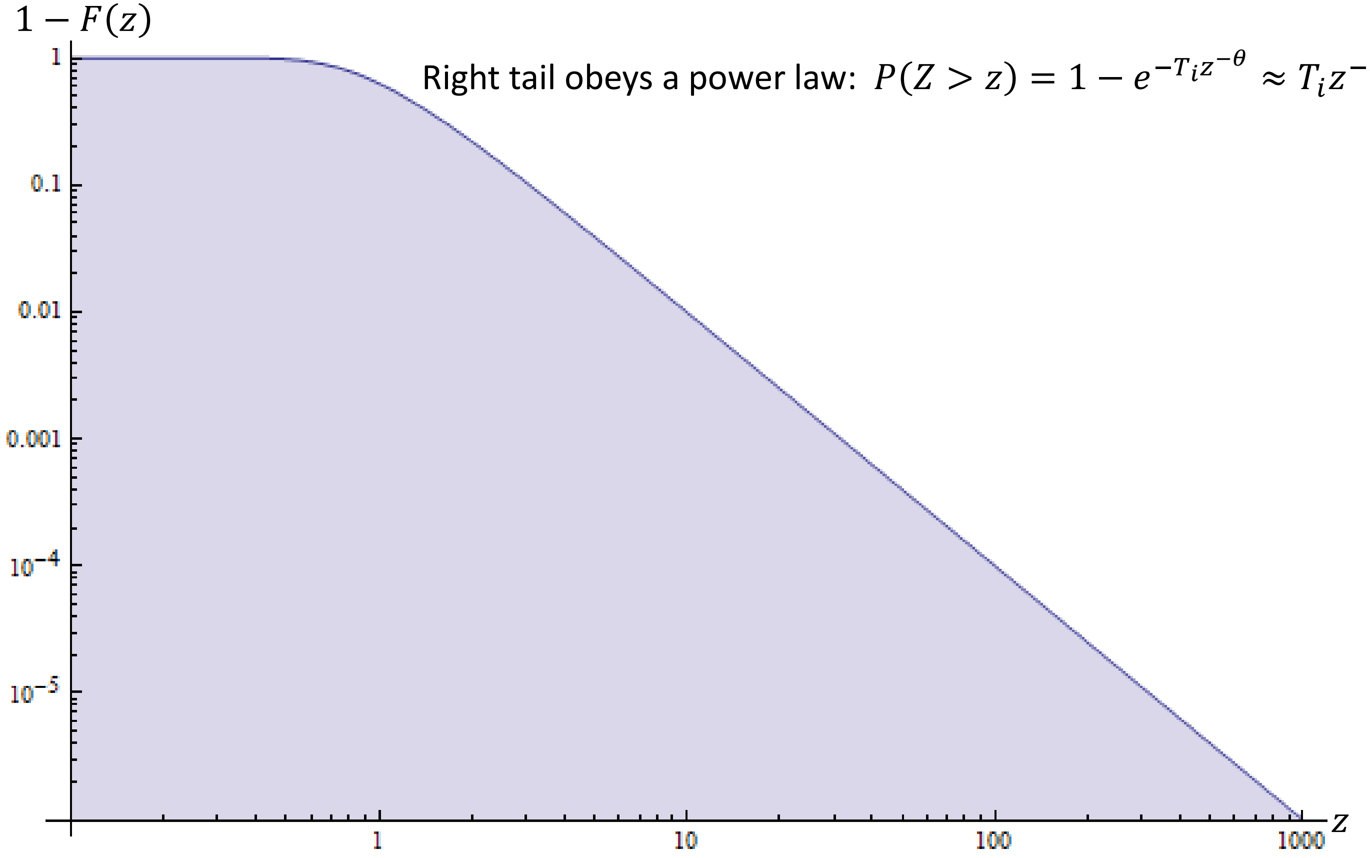
$T_i = 1$

Density of Frechet Distribution



$\theta = 2$

LogLog Plot of CCDF of Frechet Distribution



Right tail obeys a power law: $P(Z > z) = 1 - e^{-T_i z^{-\theta}} \approx T_i z^{-\theta}$ for large z

$\theta = 2, T_i = 1$

Distribution of Prices

Distribution of prices offered by firms in country i governed by productivity distribution

- Define $G_{ni}(p)$ as the proportion of prices offered by country i to country n that are less than p
- Recall $p_{ni}(j) = \left(\frac{c_i}{z_i(j)}\right) d_{ni}$, therefore

$$G_{ni}(p) = \Pr(P_{ni} < p) = \Pr\left(Z > \frac{c_i}{p} d_{ni}\right) = 1 - F_i(c_i d_{ni}/p) = 1 - e^{-T_i (c_i d_{ni})^{-\theta} p^{\theta}}$$

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Lowest price in country n will be p_n such that $p_{ni} \geq p_n \; \forall i$ (with equality for one i).

- Let $G_n(p)$ be share of (minimal) prices offered in country n that are less than p

$$G_n(p) = 1 - \prod_{i=1}^{N} \left(1 - G_{ni}(p)\right) = 1 - \prod_{i=1}^{N} e^{-T_i (c_i d_{ni})^{-\theta} p^{\theta}} = 1 - e^{-\left(\sum_{i=1}^{N} T_i (c_i d_{ni})^{-\theta}\right) p^{\theta}}$$

Distribution of Prices

Can write the proportion of prices less than p in country n as

$$G_n(p) = 1 - e^{-\Phi_n p^\theta}, \qquad \text{where } \Phi_n = \sum_{i=1}^{N} T_i (c_i d_{ni})^{-\theta}$$

Φ_n is a country specific price parameter

- T_i indexes how productive country i is (on average)
- c_i is how costly the inputs are in country i
- d_{ni} is how expensive (iceberg costs) it is to ship output from country i to country n

Distribution of Prices

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Φ_n is a country specific price parameter

- Only reason Φ_n differs across countries is due to differences in iceberg costs (d_{ni})
- Note that model can handle autarky easily: $d_{ni} = \infty \ \forall i \neq n \Rightarrow \ \Phi_n = T_n c_n^{-\theta}$

Location of Lowest Cost Producers

The probability that country i is the lowest cost producer of good j to country n, π_{ni}, is

$$\pi_{ni} = \int_0^\infty \overbrace{\prod_{\substack{s=1 \\ s\neq i}}^{N} \Pr(P_{ns} > p)}^{\text{Prob. no other country offers a price less than } p} \overbrace{d\Pr(P_{ni} \leq p)}^{\text{Prob. country } i \text{ offers a price} \leq p}$$

$$= \int_0^\infty \prod_{\substack{s=1 \\ s\neq i}}^{N} [1 - G_{ns}(p)] dG_{ni}(p)$$

$$= \int_0^\infty \prod_{\substack{s=1 \\ s\neq i}}^{N} \left[e^{-T_s(c_s d_{ns})^{-\theta} p^\theta}\right] \left[\left(T_i(c_i d_{ni})^{-\theta} \theta p^{\theta-1}\right) e^{-T_i(c_i d_{ni})^{-\theta} p^\theta}\right] dp$$

Location of Lowest Cost Producers

$$\pi_{ni} = \int_0^\infty \prod_{\substack{s=1 \\ s\neq i}}^{N} \left[e^{-T_s(c_s d_{ns})^{-\theta} p^\theta}\right] \left[\left(T_i(c_i d_{ni})^{-\theta} \theta p^{\theta-1}\right) e^{-T_i(c_i d_{ni})^{-\theta} p^\theta}\right] dp$$

$$= \left(T_i(c_i d_{ni})^{-\theta}\right) \int_0^\infty \prod_{s=1}^{N} \left[e^{-T_s(c_s d_{ns})^{-\theta} p^\theta}\right] \theta p^{\theta-1} dp$$

$$= \left(T_i(c_i d_{ni})^{-\theta}\right) \int_0^\infty e^{-\Phi_n p^\theta} \theta p^{\theta-1} dp\,; \text{ where } \Phi_n = \sum_{s=1}^{N} T_s(c_s d_{ns})^{-\theta}$$

$$= \left(T_i(c_i d_{ni})^{-\theta}\right) \left[-\frac{1}{\Phi_n} e^{-\Phi_n p^\theta}\right]_{p=0}^{\infty} = \left(T_i(c_i d_{ni})^{-\theta}\right) \left[0 - \frac{1}{\Phi_n} e^0\right]$$

$$\Rightarrow \quad \pi_{ni} = \frac{T_i(c_i d_{ni})^{-\theta}}{\Phi_n} = \frac{T_i(c_i d_{ni})^{-\theta}}{\sum_{s=1}^{N} T_s(c_s d_{ns})^{-\theta}}$$

Connection between Origin and Price Distribution

Note that $\Pr(P_n \leq \bar{p}) = \Pr(P_{ni} \leq \bar{p} | P_n = P_{ni})$ (i.e. distribution of prices in n doesn't change conditional on knowing the lowest cost producer for country n).

To see this note Bayes rule, therefore

$$\Pr(P_{ni} \leq \bar{p} | P_n = P_{ni}) = \frac{\Pr(P_{ni} \leq \bar{p}) \Pr(P_{ni} = P_i | P_{ni} \leq \bar{p})}{\Pr(P_n = P_{ni})}$$

So therefore $\Pr(P_n \leq \bar{p}) = \Pr(P_{ni} \leq \bar{p} | P_n = P_{ni})$ is the same as writing

$$G_n(\bar{p}) = \frac{1}{\pi_{ni}} \int_0^{\bar{p}} \prod_{\substack{s=1 \\ s \neq i}}^{N} [1 - G_{ns}(p)] dG_{ni}(p)$$

And the RHS simplifies to

$$= \frac{1}{\pi_{ni}} \left(T_i (c_i d_{ni})^{-\theta}\right) \left[-\frac{1}{\Phi_n} e^{-\Phi_n p^\theta}\right]_{p=1}^{\bar{p}} = \frac{1}{\pi_{ni}} \left(\frac{T_i (c_i d_{ni})^{-\theta}}{\Phi_n}\right) \left[-e^{-\Phi_n \bar{p}^\theta} - (-1)\right] = 1 - e^{-\Phi_n \bar{p}^\theta} = G_n(\bar{p})$$

CES Price Index

Note that the CES Price index can be derived as

$$p_n = \left(\int_0^1 \left(p_n(j)\right)^{1-\sigma} dj\right)^{\frac{1}{1-\sigma}} = \left(\int_0^\infty (p)^{1-\sigma} dG_n(p)\right)^{\frac{1}{1-\sigma}}$$

$$= \left(\int_0^\infty (p)^{1-\sigma} \Phi_n e^{-\Phi_n p^\theta} \theta p^{\theta-1} dp\right)^{\frac{1}{1-\sigma}}$$

Which, for $\sigma < 1 + \theta$, can simplify to (requires a change of variables in integral to $x \equiv p^\theta \Phi_j$)

$$p_n = \left[\Gamma\left(\frac{\theta + 1 - \sigma}{\theta}\right)\right]^{\frac{1}{1-\sigma}} (\Phi_n)^{-\frac{1}{\theta}} = \gamma(\Phi_n)^{-\frac{1}{\theta}}$$

Where $\Gamma[t]$ is the Gamma function

$$\Gamma(t) = \int_0^\infty x^{t-1} e^{-x} dx$$

Aggregate Bilateral Trade Flows

Model doesn't pin down which specific goods are traded

Can compute fraction of country n's expenditure on goods from country i

- Recall distribution of prices are independent of origin of lowest cost producer
- Therefore average expenditure per good does not depend on origin of good

$$\frac{X_{ni}}{X_n} = \pi_{ni} = \frac{T_i(c_i d_{ni})^{-\theta}}{\sum_{s=1}^{N} T_s(c_s d_{ns})^{-\theta}}$$

Deriving Aggregate Trade Flows

Note that country i's total production is

$$Q_i = \sum_{m=1}^{N} X_{mi} = \sum_{m=1}^{N} \frac{T_i (c_i d_{mi})^{-\theta}}{\sum_{s=1}^{N} T_s (c_s d_{ms})^{-\theta}} X_m = T_i c_i^{-\theta} \sum_{m=1}^{N} \frac{(d_{mi})^{-\theta}}{\sum_{s=1}^{N} T_s (c_s d_{ms})^{-\theta}} X_m$$

Therefore, substituting $\Phi_m = \sum_{s=1}^{N} T_s (c_s d_{ms})^{-\theta}$ and rearranging above yields

$$T_i c_i^{-\theta} = \frac{Q_i}{\left(\sum_{m=1}^{N} \frac{(d_{mi})^{-\theta}}{\Phi_m} X_m \right)}$$

Substituting in that $\Phi_n = (p_n)^{-\theta} \gamma^{\theta}$, get

$$T_i c_i^{-\theta} = \gamma^{\theta} \frac{Q_i}{\left(\sum_{m=1}^{N} \left(\frac{d_{mi}}{p_m} \right)^{-\theta} X_m \right)}$$

Deriving Aggregate Trade Flows

Still have

$$X_{ni} = \frac{T_i (c_i d_{ni})^{-\theta}}{\Phi_n} X_n$$

Plugging in previous expression for $T_i c_i^{-\theta}$ and $\Phi_n = (p_n)^{-\theta} \gamma^{\theta}$ yields

$$X_{ni} = \gamma^{\theta} \frac{Q_i}{\left(\sum_{m=1}^{N} \left(\frac{d_{mi}}{p_m} \right)^{-\theta} X_m \right)} \frac{(d_{ni})^{-\theta}}{(p_n)^{-\theta} \gamma^{\theta}} X_n$$

Which simplifies to

$$X_{ni} = \frac{\left(\frac{d_{ni}}{p_n} \right)^{-\theta} X_n}{\left(\sum_{m=1}^{N} \left(\frac{d_{mi}}{p_m} \right)^{-\theta} X_m \right)} Q_i$$

Gravity in Trade Flows

Taking log of previous equation yields

$$\log X_{ni} = \overbrace{-\log\left(\sum_{m=1}^{N}\left(\frac{d_{mi}}{p_m}\right)^{-\theta} X_m\right)}^{\text{country } i \text{ fixed effect}} + \overbrace{\theta \log p_n}^{\text{country n fixed effect}} - \overbrace{\theta \log d_{ni}}^{\text{distance between } i \text{ and } j} + \overbrace{\log X_n}^{\text{country } n'\text{s size}} + \overbrace{\log Q_i}^{\text{country } i'\text{s size}}$$

- Generates a gravity equation similarly to the Armington framework
- Can run a gravity regression and use that to pin down θ if had distance data

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Next lecture: closing the model to determine equilibrium input costs (e.g. wages) + counterfactuals