ECO 330: Economics of Development Professor Jack Rossbach

Growth Accounting Notes

Adapted from notes by Professor Tim Kehoe

Overview

Growth Accounting decomposes Output per Worker into three factors: Productivity, Capital, and Labor. The decomposition works by manipulating a Cobb-Douglas aggregate production function of the form:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

where Y_t is output, A_t is productivity or TFP (Total Factor Productivity), K_t is Capital Input, and L_t is Labor Input. The parameter α represents Capital's share of Output, while $(1-\alpha)$ represents Labor's share of Output. Since the two shares sum up to 1, this production function features Constant Returns to Scale, which means that if all inputs (Capital and Labor) are doubled, then Output will double as well.

Decomposition

To decompose Output per Worker into our three components we follow three steps.

Starting with the aggregate production function above.

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

Step 1: Divide Both Sides by $(Y_t)^{\alpha}$ and simplify terms

$$Y_t^{1-\alpha} = A_t \left(\frac{K_t}{Y_t}\right)^{\alpha} L_t^{1-\alpha}$$

Step 2: Raise both sides to the power $1/(1-\alpha)$

$$Y_t = (A_t)^{\frac{1}{1-\alpha}} \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} L_t$$

Step 3: Divide both sides by the number of Workers, N_t,

$$\frac{Y_t}{N_t} = (A_t)^{\frac{1}{1-\alpha}} \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} \frac{L_t}{N_t}$$

Which is the formula we use for our growth decomposition. Y_t/N_t is Output per Worker, $(A_t)^{\frac{1}{1-\alpha}}$ is the Productivity factor, $(K_t/Y_t)^{\frac{\alpha}{1-\alpha}}$ is the Capital factor, and depends on the Capital-Output ratio, and L_t/N_t is Labor Supplied per Worker and is the Labor factor.

Total Factor Productivity (TFP) can be estimated using the production function according to

$$A_t = \frac{Y_t}{K_t^{\alpha} L_t^{1-\alpha}}$$

Estimating the Capital Stock

National Accounts typically do not have data on Capital Stocks. For this reason, it is necessary to estimate the Capital Stock, using data on Investment (which is a flow, rather than a stock) and the Perpetual Inventory Method. This method works by guessing an initial Capital Stock K_0, and then using the Law of Motion for Capital,

$$K_{t+1} = (1-\delta)K_t + I_t,$$

to estimate the capital stock for future periods, where δ is the depreciation rate for capital and I_t is investment.

For this method, it is important to have a reasonable guess for the initial capital stock, but having a perfect guess is not necessary. The reason a perfect initial guess is not necessary, is that after T years, only $(1-\delta)^T$ of the initial capital stock is still around — the rest having depreciated. That means that as long as your initial guess was reasonable, as time passes, the difference between the actual capital stock and your estimated capital stock will become negligible.

How do we make sure our initial guess is reasonable? Most methods deliver relatively similar results for developed countries (when countries are in stages of extreme transition, guessing an initial capital stock is trickier). The method we will use in this class is to choose an initial capital stock such that the Capital-Output Ratio in the first year is equal to the average Capital-Output Ratio over the next 10 years:

$$\frac{K_0}{Y_0} = \frac{1}{10}\sum_{t=1}^{10}\frac{K_t}{Y_t}$$

Since changing K_0 changes $K_1, K_2, \ldots$, we need to use a numerical solver to find this value. In Excel, this can be done using the solver add-in or Data→What-if-Analysis→Goal Seek:

$$\textbf{Goal Seek:}\ \text{Choose } \frac{K_0}{Y_0} \text{ such that } \left|\frac{K_0}{Y_0} - \frac{1}{10}\sum_{t=1}^{10}\frac{K_t}{Y_t}\right| = 0$$

Indexing

When doing Growth Accounting, we are most interested in observing changes over time. For that reason, we often choose to report and graph our Growth Decomposition indexed to a base year. The index is computed by dividing the value of each factor, by the factor's value in a given base year and multiplying by 100:

$$\text{Indexed Value} = 100 \times \left(\frac{\text{Value}}{\text{Value in Base Year}}\right)$$

This puts all the variables on a common scale, and allows us to easily observe how much each factor has changed over the time period relative to its initial value in the base year.

Variables and Data Sources

Y_t (output): Real GDP. Common sources for real GDP include the Organization for Economic Co-Operation and Development's (OECD) *Annual National Accounts* and the International Monetary Fund's (IMF) *International Financial Stastistics* (IFS). Real GDP is typically referred to as GDP in Constant Prices, but can be computed using Nominal GDP, or GDP in Current Prices, deflated by the GDP Deflator. Historic Data is available from the *Maddison Project Database*, inspired by the work of Angus Maddison. Purchasing Power Parity (PPP)-adjusted data is available from the *Penn World Tables*, hosted by the University of Groningen.

L_t (labor input): total annual hours worked. The OECD has data on Hours Worked in their *Population and Labor Force Statistics*, otherwise The Conference Board and Groningen Growth Development Centre hosts the *Total Economy Database*, which has hours worked for most countries. If hours worked is not available, it can be estimated using employment rates.

N_t (working age population): population ages 15–64. Available from the OECD's *Population and Labor Force Statistics* and the World Bank's *World Development Indicators*.

$1-\alpha$ (labor's share of income). Labor income share is unambiguous labor income divided by the sum of unambiguous labor income and unambiguous capital income. Ambiguous portions of income are subtracted from GDP to get the sum of unambiguous labor income and unambiguous capital income. The labor income share can be computed as

$$1-\alpha = \frac{CE}{GDP-(OSMI-CF)-T}$$

Where CE is Compensation of Employees, OSMI is Household Gross Operating Surplus and Mixed Income, CF is Household Consumption of Fixed Capital, and T is Taxes Less Subsidies. Data is available from the OECD's *Annual National Accounts*, however data is not available for many countries over long periods. Often, a value of $1-\alpha=0.7$ (and therefore $\alpha=0.3$) is used, which tends to be close to the value we get when we compute it directly.

I_t (investment): Real Investment. Investment data is available from the OECD's *Annual National Accounts* and the IMF's *IFS.* When computing investment we sum together data on Gross Fixed Capital Formation and Changes in Inventories. Instead of using data on real investment, it is often better to collect data on nominal investment and deflate it by the GDP deflator so that capital and output are deflated by the same price index.

δ (capital depreciation rate). The OECD's *Annual National Accounts* reports Consumption of Fixed Capital, which is equal to δK_t. We often choose the value of δ to match the average ratio of Capital Depreciation to GDP over the growth accounting period. Note that our choice of δ affects our estimated capital stock through the law of motion of capital. Therefore, δ and K_0 need to be estimated jointly. For this class we will use a value of $\delta=0.05$, which tends to be close to the estimated value.