ECO 330: Economics of Development

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What Makes Countries Grow?

Common Answers

- Technological progress
- Capital accumulation

Question: Should countries converge over time?

Models of Economic Growth – Malthusian Model

Recap of Malthusian Model

- Countries don't grow (in long run) from productivity increases
- Productivity increase $\Rightarrow$ more people $\Rightarrow$ less natural resources (land) per person $\Rightarrow$ less output per person $\Rightarrow$ no change in GDP per capita
- Only way to increase GDP per capita is to implement population control
- No strong predictions for convergence/divergence

Caveat

- Applies to Agrarian economies (pre-Industrial Revolution)

Models of Economic Growth – Solow Model

Introduction to Solow Growth Model

- Growth will come from three sources: Capital, Labor, and Output
- Investment will lead to Capital accumulation over time
- Countries with more Capital per worker will be richer

Solow Growth Model: Setup

Three components to Solow Growth Model

1. Production Function
2. Consumption and Investment
3. Law of Motion for Capital

Use those elements to find Steady State Equilibrium

- Steady State means GDP per capita and Capital per Worker doesn't change

Elements of the Solow Growth Model: Production

1. Cobb-Douglas Production Function

$$Y_t = A_t(K_t)^{\alpha}(L_t)^{1-\alpha}$$

- Output (Y) is created using capital (K) and labor (L) as inputs.
- A is Total Factor Productivity (TFP). t subscript denotes the year, $\alpha \in (0,1)$ is capital intensity

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Features of production function:

- Diminishing marginal returns in each individual input (double only $K \Rightarrow$ less than double Y)
- Constant returns to scale (double both K and $L \Rightarrow$ double Y)

Numerical Example

- Let $A_t = 2$, $\alpha = 0.3$, Production function becomes

$$Y = 2(K_t)^{0.3}(L_t)^{0.7}$$

Suppose $K = 15$, $L = 15$, then

$$Y = 2 \times (15)^{0.3} \times (15)^{0.7} = 2 \times 2.25 \times 6.65 = \mathbf{30}$$

What happens if we double each input?

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What happens if we double each input?

Capital Input	Labor Input	Output
15	15	30
30	15	37
15	30	49
30	30	60

Diminishing Marginal Returns in Single Factor

- Let $A_t = 2$, $\alpha = 0.3$, and fix $L = 100$
- We get less additional output for each additional unit of capital

Capital Input	Increase	Output	Increase
50	--	162	--
100	50	200	37
150	50	226	25
200	50	246	20

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If we have small capital stock, large gains from adding one more unit of capital.

- Increasing K from 10 to 11 increases output by 2.9 units.

Elements of the Solow Growth Model: Consumption/Investment

2. Consumption versus Investment

- Each unit of output can either be consumed, or saved and used as investment

$$\overbrace{Y}^{\text{Output}} = \overbrace{C}^{\text{Consumption}} + \overbrace{I}^{\text{Investment}}$$

- Investment will be used to create capital, and add to the capital stock for the next period

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- Save a constant fraction of income each year, $0<s<1$

$$I = sY, \qquad C = (1 - s)Y$$

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- **Example:** Let $s = 0.05$ (save 5% of output) and $Y = 200$. Then $I = 10$ and $C = 190$

Elements of the Solow Growth Model: Capital Accumulation

2. Law of Motion for Capital

- A constant fraction, 0<δ<1, of the capital stocks depreciates every year
- Investment gets added to next years capital stock

$$\overbrace{K_{t+1}}^{\substack{\text{Capital Stock}\\ \textbf{Next}\text{ Year}}} = \overbrace{K_t}^{\substack{\text{Capital Stock}\\ \textbf{This}\text{ Year}}} - \overbrace{\delta K_t}^{-\text{Depreciation}} + \overbrace{I_t}^{+\text{Investment}}$$

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Capital Accumulation: Numerical Example

Let $\delta = 0.05$ (5% of capital stock depreciates annually)

- Suppose $K_t = 100$ and $I_t = 10$
- According to **Law of Motion for Capital**

$$K_{t+1} = (1 - 0.05) \times 100 + 10 = 95 + 10 = \mathbf{105}$$

- The capital stock increases

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What if $\delta = 0.10$? Then

$$K_{t+1} = (1 - 0.10) \times 100 + 10 = 90 + 10 = \mathbf{100}$$

- Here, the capital stock stays the same

Combining the Pieces: GDP per Capita

We want to see what the model implies for Output (GDP) per Capita

- Suppose each person has 1 unit of labor ⇒ Population = Labor Supply
- From the production function, GDP per Capita (Y_t/L_t) is

$$\frac{Y_t}{L_t} = \frac{A_t(K_t)^{\alpha}(L_t)^{1-\alpha}}{L_t} \Rightarrow \frac{Y_t}{L_t} = A_t\left(\frac{K_t}{L_t}\right)^{\alpha}$$

where (K_t/L_t) is capital per worker.

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where (K_t/L_t) is capital per worker.

Since $\alpha < 1$, there are diminishing marginal returns from capital per person

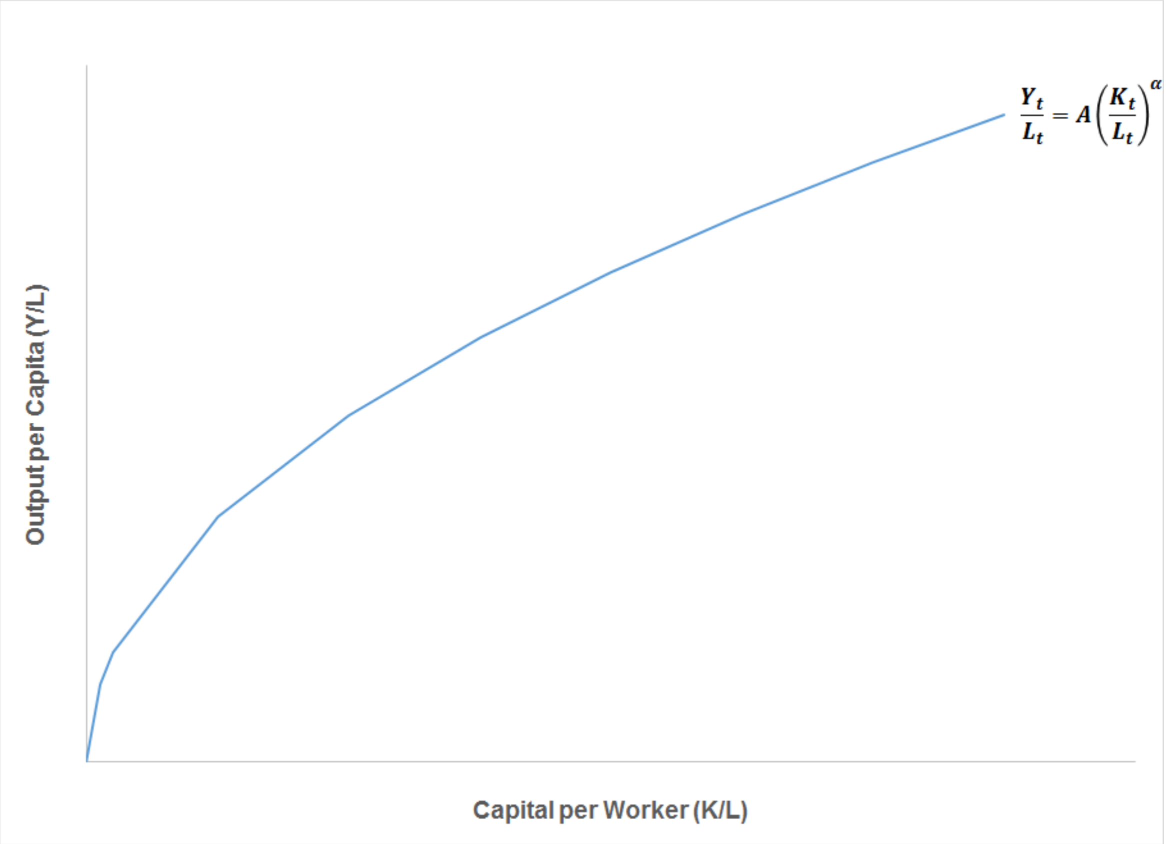
Diminishing Marginal Returns in Capital per Worker

- Let $A_t = 2$, $\alpha = 0.3$, and fix $L = 100$
- We get less additional output for each additional unit of capital

K/L	Increase	Y/L	Increase
0.5	--	1.62	--
1.0	0.5	2.00	0.37
1.5	0.5	2.26	0.25
2.0	0.5	2.46	0.20

Important: Increasing Capital per worker always increases GDP per Capita

Diminishing Marginal Returns in Capital per Worker



Capital Accumulation and Growth

For simplicity, assume Population and Technology stay constant

From Law of Motion of Capital we have

$$K_{t+1} = (1-\delta)K_t + \overbrace{sY_t}^{I_t}$$

- Dividing both sides by labor gives

$$\frac{K_{t+1}}{L_t} = (1-\delta)\frac{K_t}{L_t} + s\frac{Y_t}{L_t}$$

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$$\frac{K_{t+1}}{L_t} = (1-\delta)\frac{K_t}{L_t} + s\frac{Y_t}{L_t}$$

- Multiplying both sides by L_t/L_{t+1} gives expression for accumulation of Capital per Worker

$$\frac{K_{t+1}}{L_{t+1}} = \frac{L_t}{L_{t+1}}\left[(1-\delta)\frac{K_t}{L_t} + s\frac{Y_t}{L_t}\right]$$

Capital Accumulation and Growth

The dynamics of the Solow Growth Model are governed by two equations

- Accumulation of Capital per Worker

$$\frac{K_{t+1}}{L_{t+1}} = \frac{L_t}{L_{t+1}}\left[(1-\delta)\frac{K_t}{L_t} + s\frac{Y_t}{L_t}\right], \qquad (1)$$

- Output per Worker

$$\frac{Y_t}{L_t} = A_t\left(\frac{K_t}{L_t}\right)^{\alpha}, \qquad (2)$$

Capital Accumulation and Growth: Numerical Example

Let $A_t = 2$, $\alpha = 0.3$, $\delta = 0.05$, and $s = 0.05$. Further suppose no population growth, so $\frac{L_{t+1}}{L_t} = 1$

- If we start with $K_0/L_0 = 0.5$, then Output per Worker is

$$\frac{Y_0}{L_0} = 2(0.5)^{0.3} = 1.62$$

- We can plug that in to find Capital per Worker in next period

$$\frac{K_1}{L_1} = (1)[(1 - 0.05)(0.5) + (0.05)(1.62)] = 0.556$$

- Capital per Worker increased by 0.556 units (over 10 percent) in a single year

Capital Accumulation and Growth: Numerical Example

Let $A_t = 2$, $\alpha = 0.3$, $\delta = 0.05$, and $s = 0.05$. Further suppose no population growth, so $\frac{L_{t+1}}{L_t} = 1$

- Suppose we start with Capital per Worker (K/L) equal to **0.50**, then

Time	K/L	% Increase	Y/L	% Increase
0	**0.50**	--	1.62	--
1	0.56	11.2	1.67	3.2
2	0.61	10.0	1.73	2.9
3	0.66	9.1	1.77	2.6
4	0.72	8.3	1.81	2.4

Note: % Increase is **Percentage Change** not flat increase

Capital Accumulation and Growth: Numerical Example

Let $A_t = 2$, $\alpha = 0.3$, $\delta = 0.05$, and $s = 0.05$. Further suppose no population growth, so $\frac{L_{t+1}}{L_t} = 1$

- Suppose we start with Capital per Worker (K/L) equal to **4.00**, then

Time	K/L	% Increase	Y/L	% Increase
0	**4.00**	--	3.03	--
1	3.95	−1.21	3.02	−0.36
2	3.91	−1.18	3.01	−0.36
3	3.86	−1.15	3.00	−0.35
4	3.77	−1.12	2.99	−0.34

Note: % Increase is **Percentage Change** not flat increase

Steady State Equilibrium

Steady State Equilibrium is where Y/L and K/L don't change

- For simplicity, assume Population and Technology stay constant
- This means in the steady state we should have

$$\frac{K}{L} = \left[(1-\delta)\frac{K}{L} + s\frac{Y}{L}\right], \qquad (1^*)$$

$$\frac{Y}{L} = A\left(\frac{K}{L}\right)^{\alpha}, \qquad (2^*)$$

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$$\frac{Y}{L} = A\left(\frac{K}{L}\right)^{\alpha}, \qquad (2^*)$$

- Plugging (2^*) into (1^*) and rearranging gives steady state Capital per worker

$$\frac{K}{L} = (1-\delta)\frac{K}{L} + s\left(A\left(\frac{K}{L}\right)^{\alpha}\right) \Rightarrow \frac{K}{L} = \left(A\frac{s}{\delta}\right)^{\frac{1}{1-\alpha}}$$

Steady State Equilibrium

We can use SS Capital per Worker to find SS Output per Worker

$$\frac{Y}{L} = A^{\frac{1}{1-\alpha}} \left(\frac{s}{\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

What does this tell us about growth?

- Capital accumulation can be a driver of growth, but will eventually reach SS
- Eventually, countries will stop growing without technological progress
- Savings rate, s, may be important determinant of growth and long-run wealth

Higher Savings Rate ⇒ Higher Capital-Output Ratio

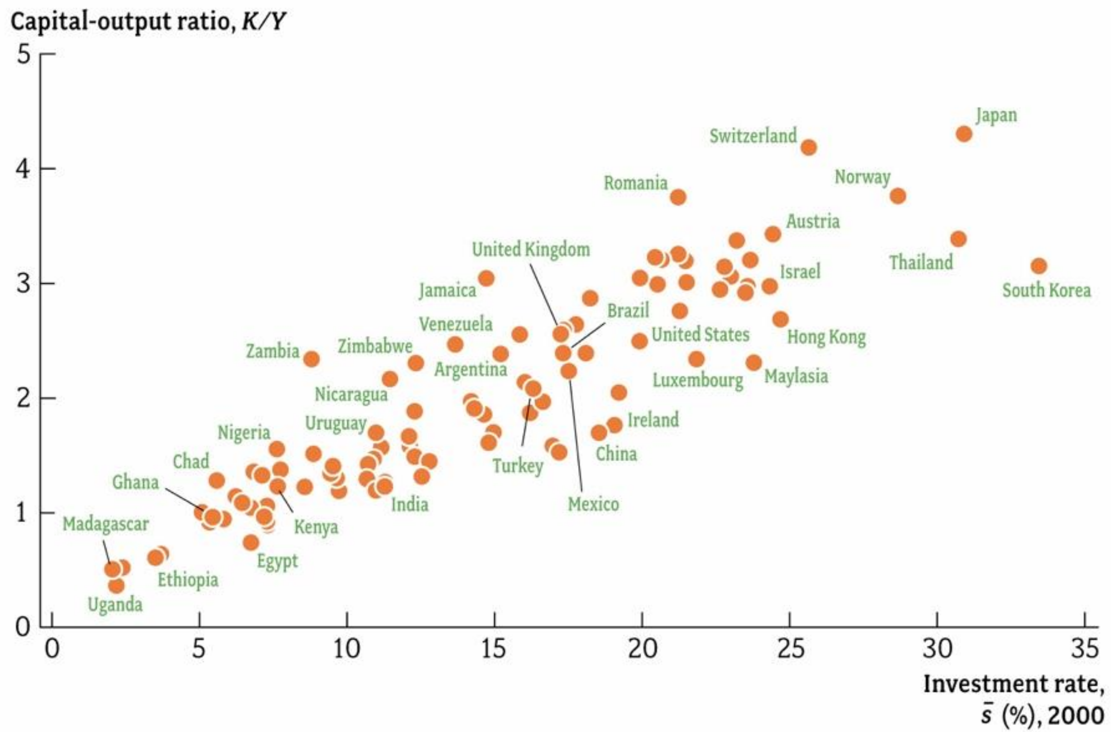
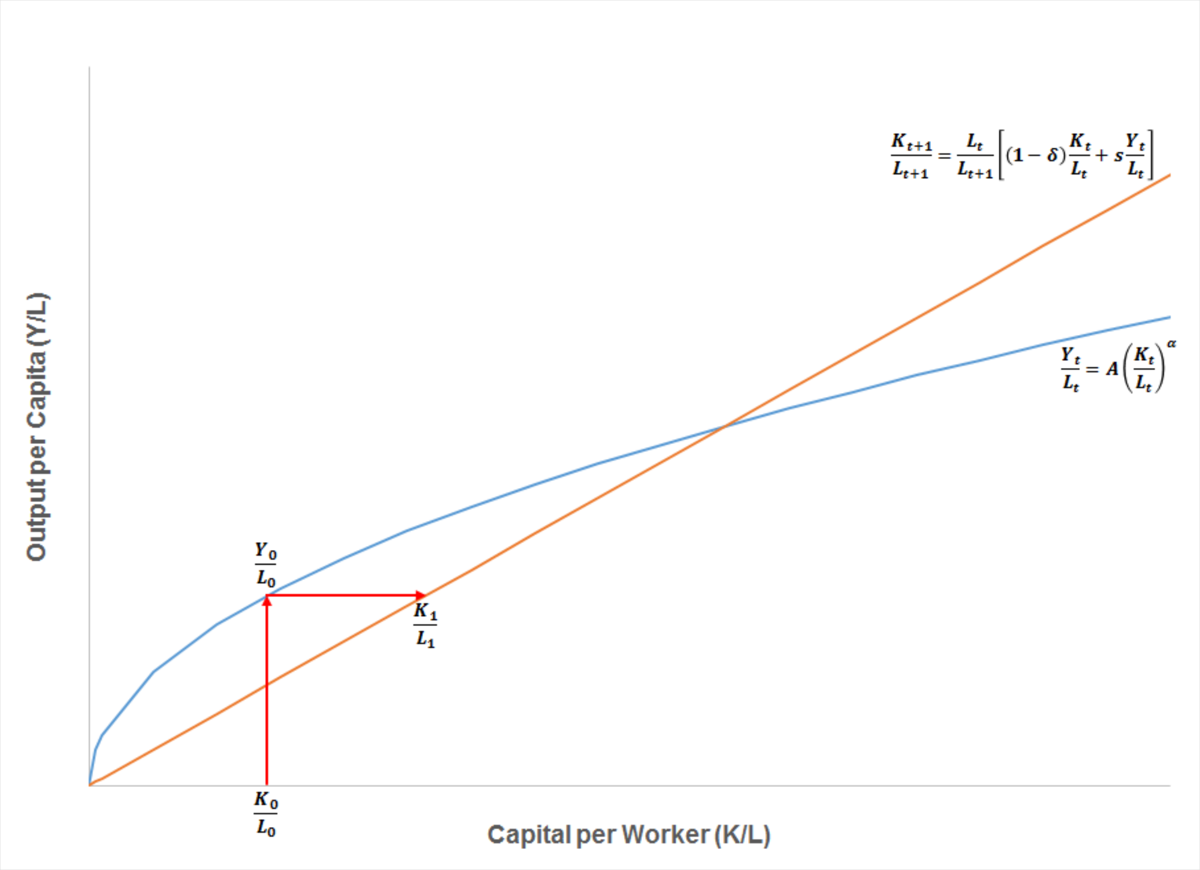


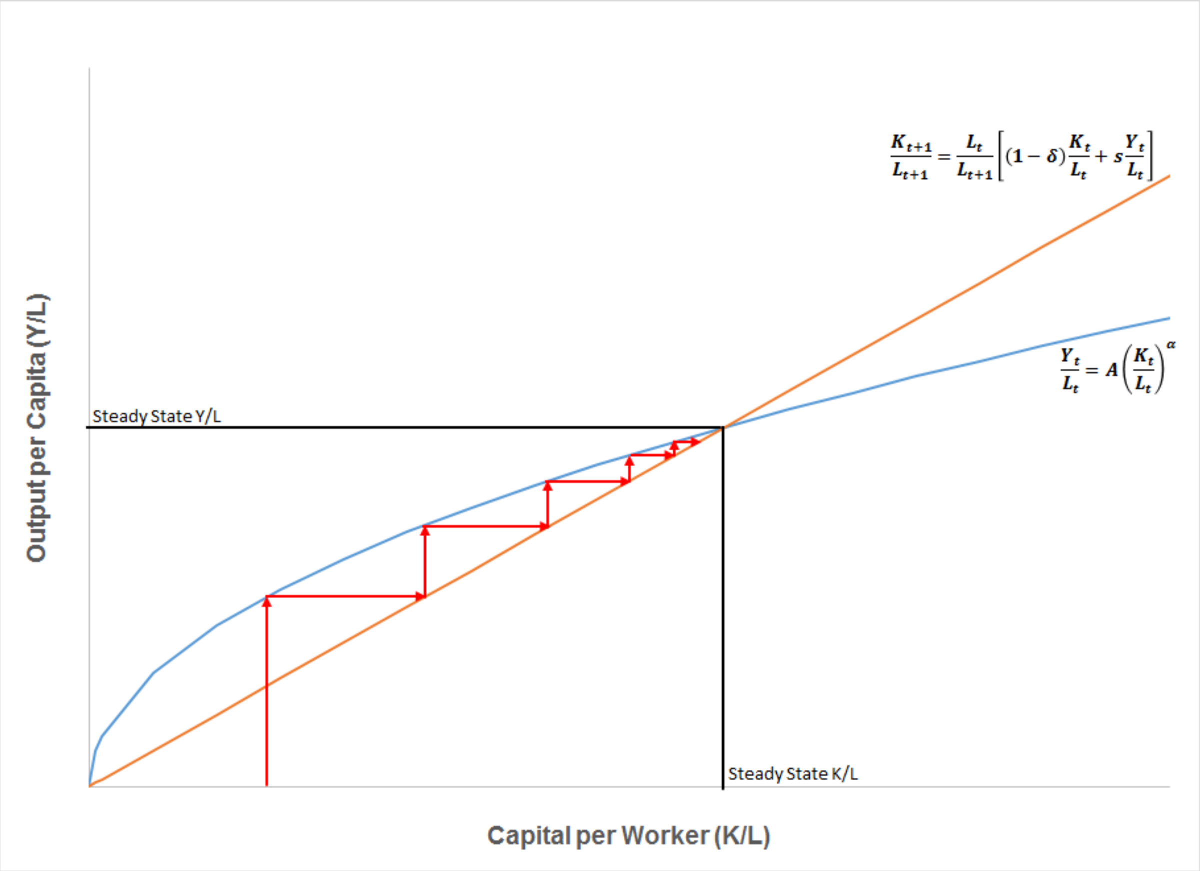
FIGURE 5.3 Explaining Capital in the Solow Model

Macroeconomics, Charles I. Jones

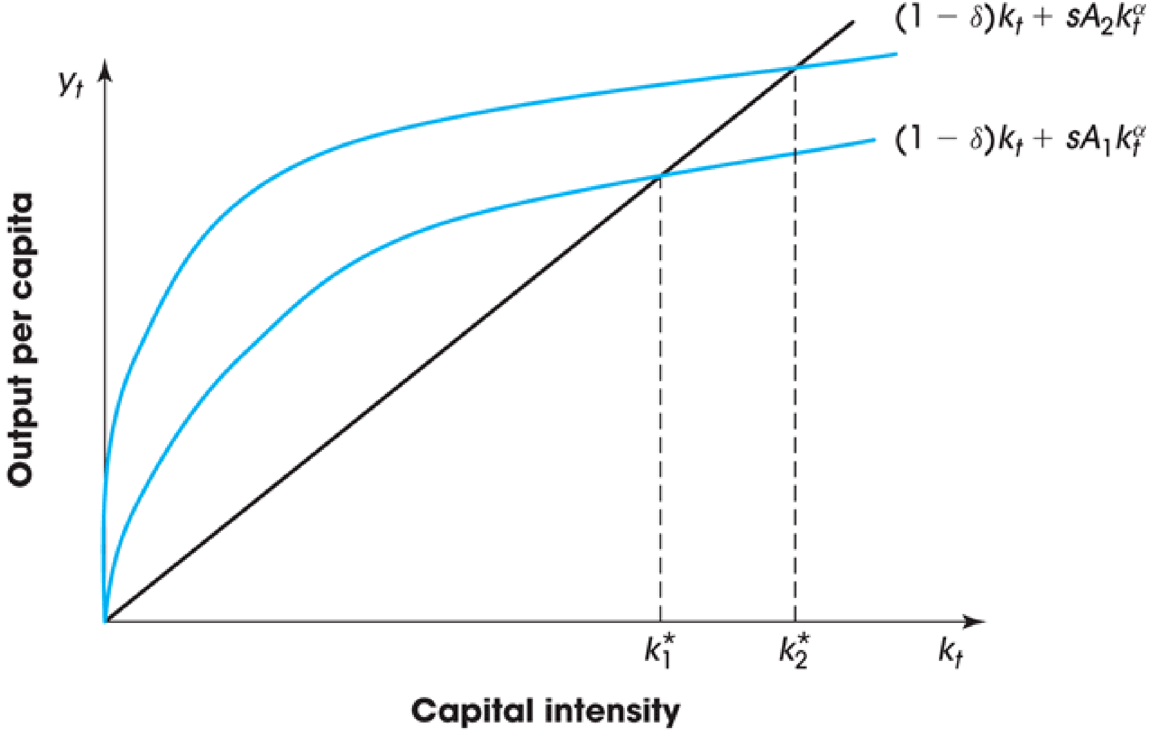
Capital Accumulation and SS Equilibrium



Capital Accumulation and SS Equilibrium



Effect of Technological Progress on SS Equilibrium



(textbook says Capital Intensity instead of Capital per Worker)

Technological progress in the economy leads to a shift from A_1 to A_2, which leads to an increase in steady state capital intensity from k_1^* to k_2^*.

Growth and Convergence

Strong predictions towards convergence

- Diminishing marginal returns $\Rightarrow$ poor countries accumulate capital faster $\Rightarrow$ should catch up

Strong predictions on whether countries can become richer

- Short run: Yes, with capital accumulation
- Long run: Only with sustained technological progress

Next steps: **Growth Accounting** and extensions/implications of Solow Growth model

No Convergence for World

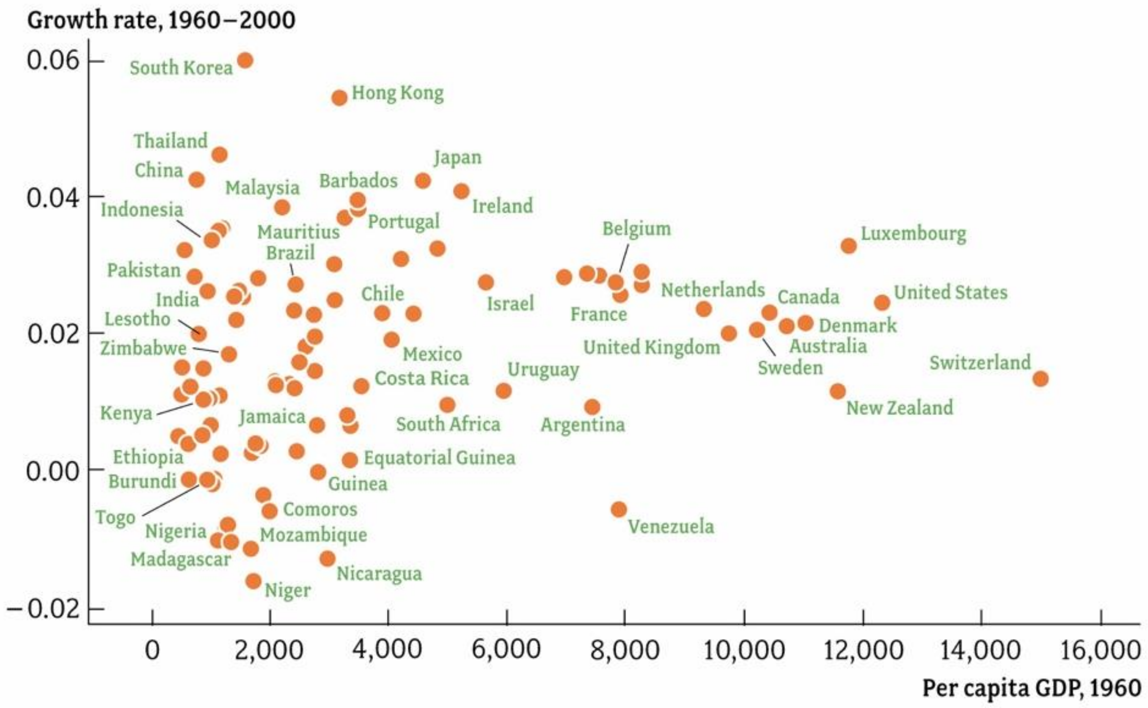


FIGURE 5.9 Growth Rates around the World, 1960–2000

Possible Convergence for OECD Countries?

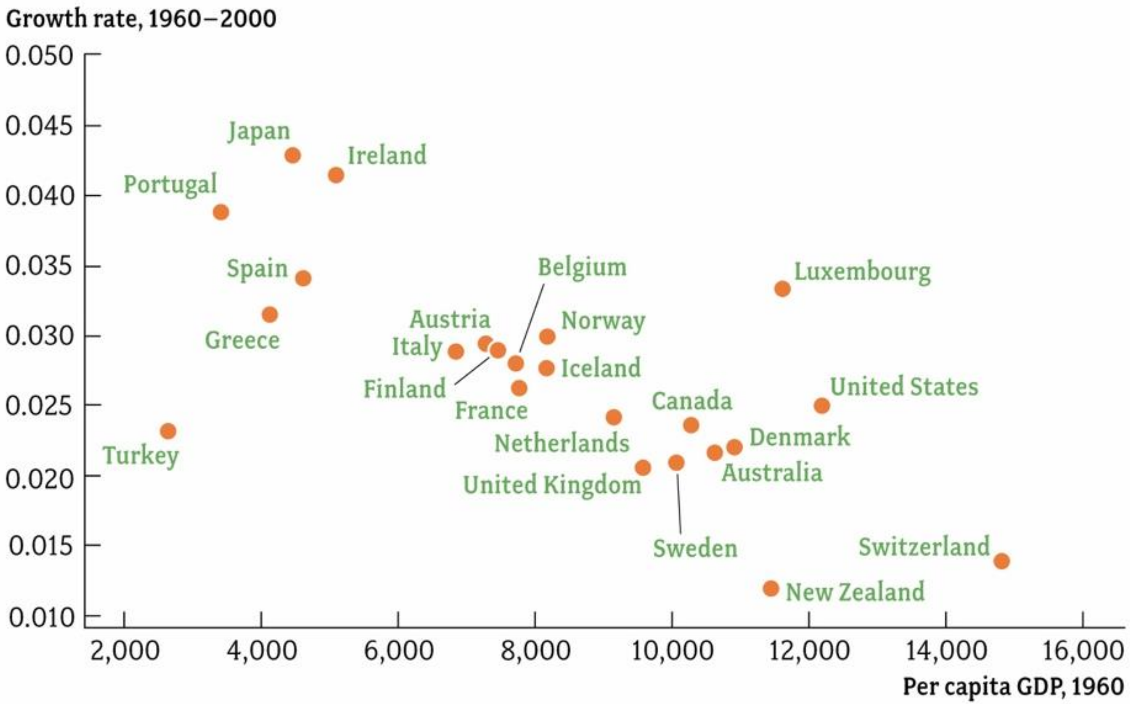


FIGURE 5.8 Growth Rates in the OECD, 1960–2000