

Cheat Sheet: Interpreting Regressions (Page 1 of 3)

Type	Transformation	X Variable	Y Variable	Equation	Interpretation of β	Intuition or Purpose	
OLS	None	Continuous	Continuous	$Y = \alpha + \beta x + \epsilon$	β : the effect of 1 unit Δ in x on Δ in y	◦ Basic regression to estimate relationships	
OLS	Log-Linear	Continuous	Continuous	$\ln(Y) = \alpha + \beta x + \epsilon$	β : the effect of 1 unit Δ in x on % Δ in y	◦ Easier interpretation, non-linear	
OLS	Linear-Log	Continuous	Continuous	$Y = \alpha + \beta \ln(x) + \epsilon$	β : the effect of 1 % Δ in x on Δ y	◦ Easier interpretation, non-linear	
OLS	Log-Log	Continuous	Continuous	$\ln(Y) = \alpha + \beta \ln(x) + \epsilon$	β : the effect of 1 % Δ in x on % Δ in y	◦ Interpret as an elasticity, non-linear	
OLS	None	Continuous	Dummy (Y = 0 or 1)	$Y = \alpha + \beta x + \epsilon$	β : the effect of 1 unit Δ in x on probability that Y occurs (a %)	◦ Think of Y as switched on and off	
OLS	None	Dummy (X = 0 or 1)	Continuous	$Y = \alpha + \beta x + \epsilon$	β : the effect of x being equal to 1 on Δ in y	◦ Think of X as switched on and off	
OLS	None	Dummy (X = 0 or 1)	Dummy (Y = 0 or 1)	$Y = \alpha + \beta x + \epsilon$	β : the effect of x being equal to 1 on probability that Y occurs (a %)	◦ Think of X and Y as switched on and off	
2SLS	Any	Continuous or Dummy	Continuous or Dummy	$Y = \alpha + \beta Z + \epsilon$ where $\text{Corr}(Z, X) > 0$ and $\text{Corr}(Z, \epsilon) = 0$	β : the <i>causal</i> effect of 1 unit Δ in x on Δ in y	◦ Causal interpretation of our β coefficient	
General Notes					Legend:		
<p>(1) With categorical values, often we create dummy variables (0/1). This makes it easier to interpret coefficients.</p> <p>(2) Many of the concepts apply across types. (i.e. how to interpret a coefficient when we have dummy Y for interaction).</p> <p>(3) This table is meant to be compact, and therefore does not include all possible iterations.</p>					X - independent variable	Δ - change	OLS - ordinary least squares
					Y - dependent variable	ln - natural log	2SLS - two stage least squares
					β - slope	log - logarithm	MLE - maximum likelihood estimator
					α - intercept	lin - linear	
					ϵ - error		

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Type	Transformation	X Variable	Y Variable	Equation	Interpretation of β	Intuition or Purpose
Any	Multivariate	Continuous	Continuous	$Y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \epsilon$	β_1 : the effect of a 1 unit Δ in x_1 on Δ in y , controlling for the effect of x_2 β_2 : the effect of a 1 unit Δ in x_2 on Δ in y , controlling for the effect of x_1	<ul style="list-style-type: none"> ◦ Control for variables ◦ Incorporate effect of multiple variables
		X ₁ Dummy	X ₂ Dummy		<p><u>Multivariate with Interaction:</u></p> β_1 (main effect): the effect of x_1 independent of the effect of x_2 β_2 (main effect): the effect of x_2 independent of the effect of x_1 β_3 (interaction): the additional marginal effect of x_1 or x_2 depending on the level of the other interacted variable	<ul style="list-style-type: none"> ◦ A way of quantifying simultaneous relationships ◦ Interaction allows for effect of one variable to vary depending on level of another variable
Any	Multivariate with Interacton	X ₁ Continuous	Continuous	$Y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 * x_2 + \epsilon$	<p><u>X₁ Dummy and X₂ Dummy:</u></p> <ul style="list-style-type: none"> ◦ Effect of $x_1=1$ is β_1 if $x_2=0$ ◦ Effect of $x_1=1$ is $(\beta_1+\beta_3)$ if $x_2=1$ ◦ Effect of $x_2=1$ is β_2 if $x_1=0$ ◦ Effect of $x_2=1$ is $(\beta_2+\beta_3)$ if $x_1=1$ 	<p><u>X₁ Continuous & X₂ Dummy:</u></p> <ul style="list-style-type: none"> ◦ Effect of x_1 is β_1 if $x_2=0$ ◦ Effect of x_1 is $(\beta_1+\beta_3)$ if $x_2=1$ ◦ Effect of $x_2=1$ is $(\beta_2+\beta_3)$
		X ₂ Dummy				
General Notes					Legend:	
<p>(1) With categorical values, often we create dummy variables (0/1). This makes it easier to interpret coefficients.</p> <p>(2) Many of the concepts apply across types. (i.e. how to interpret a coefficient when we have dummy Y for interaction).</p> <p>(3) This table is meant to be compact, and therefore does not include all possible iterations.</p>					<p>X - independent variable</p> <p>Y - dependent variable</p> <p>β - slope</p> <p>α - intercept</p> <p>ϵ - error</p>	<p>Δ - change</p> <p>ln - natural log</p> <p>log - logarithm</p> <p>lin - linear</p>
					OLS - ordinary least squares	2SLS - two stage least squares
					MLE - maximum likelihood estimator	

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Type	Transformation	X Variable	Y Variable	Equation	Interpretation of β	Intuition or Purpose
MLE	Probit	Continuous	Dummy (Y = 0 or 1)	$L(\beta) = \prod_{i=1}^N p(Y_i) = \Phi(\beta X_i)^{Y_i} \Phi(-\beta X_i)^{1-Y_i}$	β : the effect of 1 unit Δ in x on the z-score of y under a normal distribution	<ul style="list-style-type: none"> ◦ Probit is short for <i>probability unit</i> ◦ Follows standard normal distribution ◦ Thinner tails ◦ Approaches axes more quickly ◦ Higher value of β means event is more likely
MLE	Logit	Continuous	Dummy (Y = 0 or 1)	$L(\beta) = \prod_{i=1}^N \left(\frac{\exp(\beta X_i)}{1 + \exp(\beta X_i)} \right)^{Y_i} \left(\frac{\exp(\beta X_i)}{1 + \exp(\beta X_i)} \right)^{1-Y_i}$	β : the effect of 1 unit Δ in x on the z-score of y under a logistic distribution	<ul style="list-style-type: none"> ◦ Follows logistic distribution ◦ Fatter tails ◦ Approaches axes less quickly ◦ Higher value of β means event is more likely
2SLS	Any	Continuous or Dummy	Continuous or Dummy	$Y = \alpha + \beta Z + \varepsilon$ where $\text{Corr}(Z, X) > 0$ and $\text{Corr}(Z, \varepsilon) = 0$	β : the <i>causal</i> effect of 1 unit Δ in x on Δ in y	<ul style="list-style-type: none"> ◦ Causal interpretation of our β coefficient
General Notes					Legend:	
<p>(1) With categorical values, often we create dummy variables (0/1). This makes it easier to interpret coefficients.</p> <p>(2) Many of the concepts apply across types. (i.e. how to interpret a coefficient when we have dummy Y for interaction).</p> <p>(3) This table is meant to be compact, and therefore does not include all possible iterations.</p>					<p>X - independent variable Δ - change</p> <p>Y - dependent variable ln - natural log</p> <p>β - slope log - logarithm</p> <p>α - intercept lin - linear</p> <p>ε - error</p>	