Tips for Problem Set 1

To make the algebra a little easier, I made a change in how I write preferences (This is a cosmetic change, and didn't actually change anything about the problem)

New preferences are

$$U^i(c_0^i, c_1^i, c_2^i) = \theta \log c_0^i + \left(\frac{1-\theta}{2}\right)\left(\log c_1^i + \log c_2^i\right)$$

Old preferences are

$$\theta_0 \log c_0^i + (1-\theta_0)\left(\log c_1^i + \log c_2^i\right)$$

Note they are the same since we can do monotonic transfers to utility without changing preferences. Just divide the old preferences by $\left(\theta_0 + (1-\theta_0) + (1-\theta_0)\right)$ and then substitute

$$\theta = \frac{\theta_0}{\left(\theta_0 + (1-\theta_0) + (1-\theta_0)\right)}$$

Tips for Part 1.i) Look at slide 12 of first set slides from week 3. It's the same, but with one more good.

Important: When doing the budget constraint, the price of the non-traded good may differ across countries, since non-traded (i.e. infinite trade costs). The prices of the traded goods will be the same in both countries since no trade costs.

Lagrangian with new preferences

$$\mathcal{L} = \theta \log c_0^i + \left(\frac{1-\theta}{2}\right)\left(\log c_1^i + \log c_2^i\right) - \lambda(\text{Budget Constraint})$$

I didn't put the budget constraint above since I want to you figure it out yourself.

First order Conditions are

$$[c_0^i]\text{: } \frac{\partial \mathcal{L}}{\partial c_0^i} = \frac{\theta_0}{c_0^i} - \lambda p_0^i = 0$$

$$[c_1^i]\text{: } \frac{\partial \mathcal{L}}{\partial c_1^i} = \frac{\left(\frac{1-\theta}{2}\right)}{c_1^i} - \lambda p_1 = 0$$

$$[c_2^i]\text{: } \frac{\partial \mathcal{L}}{\partial c_2^i} = \frac{\left(\frac{1-\theta}{2}\right)}{c_2^i} - \lambda p_2 = 0$$

The FOC with respect to λ just gives you back the Budget Constraint. Use the above equations to get relative consumptions (i.e. c_2^i and c_0^i in terms of c_1^i) and then substitute those into the budget constraint to find consumption for one good in terms of preference parameters, income, and price. Use the relative consumptions to get consumption for the other two goods.

Tips for Part 1.ii) See slides 37-44 from the first set of slides from week 3.

Tips for Part 1.iii) See slides 18-21 in same set as above. Careful about market clearing for traded goods vs non-traded goods.

Tips for Part 1.iv) None.

Tips for Part 1.v) See slides 13-15 in second set of slides from week 3. Don't worry about balanced trade for this problem set.

Tips for Part 1.vi) The goal is to use the equilibrium equations from above so that you have all endogenous variables in terms of only exogenous parameters. There are many ways you can do this, here is one such way.

Step 0) Normalize the home wage to 1 (you can normalize any price you want, I think this is easiest)

Step 1) Your answer to 1.iv gives us knowledge of which countries will produce which goods in equilibrium. Use this to find prices from the firms problem in part 1.ii)

Step 2) Plug prices in to find consumption for the non-traded good in terms of wages using your answer to part 1.i)

Step 3) Use consumption of non-traded good to get labor allocated towards non-traded sector. Then find labor allocated towards the traded good using labor market clearing along with your knowledge of which country will produce good 1 and which country will produce good 2.

Step 4) Use prices from step 1 to find consumption of the traded goods using 1.i)

Step 5) Use goods market clearing for good 2 to find the wage of the Foreign country. This step involves the most algebra. Careful with minus signs, e.g. $1-(1-\theta)=\theta$ not $-\theta$.

Step 6) Write every other variable in terms of only exogenous parameters and the foreign wage, which we found in step 5. You do not have plug in the parameters from step 5, the wage itself is enough.

Tips for Part 1.vii) If the denominator of a fraction decreases, that increases the value of the fraction. The terms of trade for the foreign country is p_2/p_1.

Other tips: I prefer people attempt to solve this problem algebraically, however, if you want to solve 1.vi and 1.vii using R you can do so. Just pick some parameter values, and implement a relatively small transfer so your code converges. I would start with the example code for the 2x2 model on my website and update that with the new variables and equilibrium equations.