ECO 445/545: International Trade

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Recap and Roadmap

What have we done so far?

- Simple graphical analysis of trade with comparative advantage
- Formalized the model and developed a precise definition of an equilibrium in the model
- Solved the model in autarky

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Where are we going?

- Solve the model with trade
- Circle back to the graphical analysis and compare it with our formalized model
- Start building on the model, which will allow us to do policy analysis

Longer Term

What are the ultimate goals in this class?

- Understand how to develop quantitative predictions using models
- Learn to estimate the exogeneous model parameters using data
- Start solving models computationally

Walras’ Law

Walras‘ Law (loosely), in general equilibrium models, if all but one of the equilibrium equations hold, then the last equilibrium equation will hold automatically

- Observed Walras‘ Law when solving the autarky equilibrium, as we didn‘t use labor market clearing
- Essentially means we have one less equilibrium equation than we thought (It does still have to hold in equilibrim, it just does so automatically)
- We also have one less equilibrium unknown (endogeneous variable) than we thought, since only relative prices matter for allocations. Typically normalize wage equal to 1, but it doesn‘t matter which price we normalize or what value we set it equal to
- Note: A common mistake when putting models on the computer is to forget Walras‘ law

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Solving Free Trade Equilibrium

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- Two assumption to make things easier:

1. Suppose $a_1^H / a_2^H < a_1^F / a_2^F$, so Home has comparative advantage in good 1
2. Suppose we will have complete specialization in equilibrium

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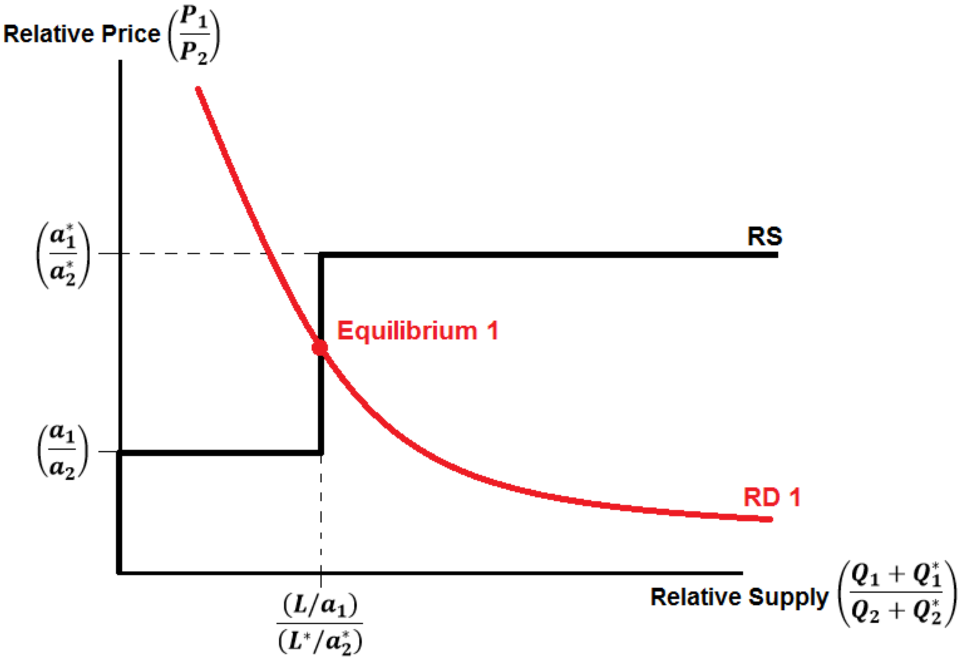
2. Suppose we will have complete specialization in equilibrium

How do we know second assumption will hold? We don't. But we can solve for implied equilibrium under complete specialization and verify it works

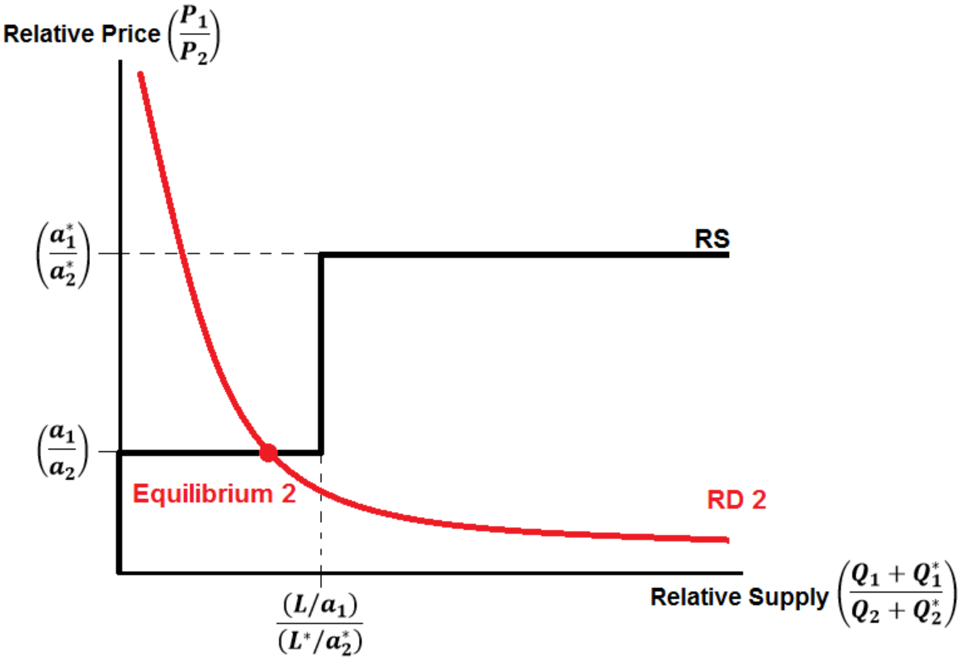
- We will get values inconsistent with an equilibrium if it doesn't

Finding Equilibrium using Relative Demand



- We solve model assuming equilibrium will be consistent with the above pattern of complete specialization.
- Afterwards we can check whether this assumption was valid (think of it as guess and check).

Finding Equilibrium using Relative Demand



- Possible that true equilibrium is supposed to be like this (Home produces both goods, foreign produces only good 2), in which case we will see our solution isn't a valid equilibrium.
- Rigorous way is to solve model for all five patterns of specialization (we don't, to save time)

Additional Simplifying Assumptions

In Economics, very important to understand your model's assumptions.

What other simplifying assumptions did we make and why?

- Countries have same preferences. **Why:** Notational simplicity. Makes the algebra slightly simpler, since each country will have same relative consumption bundles.

- Important to consider your question/goal before making assumptions. For this problem we are looking for a simple, stylized, GE model of trade we can build off of.

Equilibrium Definition

Equilibrium is prices $\{p_1, p_2\}$, wages, $\{w_H, w_F\}$ and allocations $\left\{c_1^i, c_2^i; l_1^i, l_2^i; y_1^i, y_2^i\right\}_{i\in\{H,F\}}$ s.t.

1. Consumers maximize utility

2. Firms maximize profits

3. Markets clear

Full System of Equations for Equilibrium

Equilibrium Unknowns: prices $\{p_1, p_2\}$, wages, $\{w_H, w_F\}$ and allocations $\{c_m^i; l_m^i; y_m^i\}_{\substack{i\in\{H,F\} \\ m\in\{1,2\}}}$

Equilibrium Equations

- Consumer Optimization in each country, $i = H, F$

$$c_1^i = \frac{w^i L^i}{p_1}\left(\frac{\theta_1}{\theta_1 + \theta_2}\right);\ c_2^i = \frac{w^i L^i}{p_2^i}\left(\frac{\theta_2}{\theta_1 + \theta_2}\right)$$

- Firm Optimization in each country, $i = H, F$, for each good, $m = 1,2$

$$\frac{p_m}{a_m^i} = w^i, \qquad \text{if } l_m^i > 0; \ \text{production function: } y_m^i = \frac{1}{a_m} l_m^i$$

Market clearing conditions for labor and goods on next slide

Full System of Equations for Equilibrium

Equilibrium Equations

- Labor market clearing for each country, $i = H, F$

$$l_1^i + l_2^i = L^i$$

and goods market clearing for each good

$$\overbrace{c_1^H + c_1^F}^{\text{World Consumption of Good 1}} = \overbrace{y_1^H + y_1^F}^{\text{World Production of Good 1}}$$

$$c_2^H + c_2^F = y_2^H + y_2^F$$

Full System of Equations for Equilibrium

Equilibrium Equations

- One additional constraint: **Trade is Balanced** for each country

$$\text{Value Exports} = \text{Value Imports}$$

Under of assumption of complete specialization (Home makes good 1, Foreign good 2)

$$\overbrace{p_2 c_2^H}^{\text{Value of Home Imports}} = \overbrace{p_1 c_1^F}^{\text{Value of Foreign Imports}}$$

- Note that Home Imports = Foreign Exports since only two countries
- In general, requires additional notation I don't want to introduce yet. For now, let this be the equilibrium equation that we let hold automatically due to Walras' law.

Solving Free Trade Equilibrium

Step 1: We are assuming compete specialization, so plug that into our equations

Home produces only good 1. Foreign produces only good 2.

- Market Clearing for Goods becomes

$$c_1^H + c_1^F = y_1^H$$

$$c_2^H + c_2^F = y_2^F$$

- Market Clearing for Labor becomes **equilibrium labor allocations**

$$l_1^H = L^H; \;\; l_2^F = L^F$$

- Prices must satisfy

$$p_1 = a_1^H w^H; \;\; p_2 = a_2^F w^F$$

Solving Free Trade Equilibrium

Step 2: Plug consumption from consumer problem into market clearing

- For Good 1, we have

$$\overbrace{\frac{w^H L^H}{p_1}\left(\frac{\theta_1}{\theta_1+\theta_2}\right)}^{\mathrm{c}_1^{\mathrm{H}}} + \overbrace{\frac{w^F L^F}{p_1}\left(\frac{\theta_1}{\theta_1+\theta_2}\right)}^{c_1^F} = y_1^H$$

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- Plugging in prices

$$\overbrace{\frac{w^H L^H}{a_1^H w^H}\left(\frac{\theta_1}{\theta_1+\theta_2}\right)}^{c_1^H} + \overbrace{\frac{w^F L^F}{a_1^H w^H}\left(\frac{\theta_1}{\theta_1+\theta_2}\right)}^{c_1^F} = y_1^H$$

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Collecting terms, simplifying, and rearranging

$$\left(L^H + \frac{w^F}{w^H} L^F\right)\left(\frac{\theta_1}{\theta_1+\theta_2}\right) = a_1^H y_1^H$$

Step 3: Plug **production function + labor clearing** into the above

$$\left(L^H + \frac{w^F}{w^H} L^F\right)\left(\frac{\theta_1}{\theta_1+\theta_2}\right) = a_1^H \overbrace{\left(\frac{1}{a_1^H} L^H\right)}^{y_1^H}$$

Solving Free Trade Equilibrium

$$\left(L^H + \frac{w^F}{w^H} L^F\right)\left(\frac{\theta_1}{\theta_1 + \theta_2}\right) = a_1^H \overbrace{\left(\frac{1}{a_1^H} L^H\right)}^{y_1^H}$$

Canceling the a_1^H gives an expression that only depends on **relative wages**

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Dividing both sides by $\left(\theta_1/(\theta_1 + \theta_2)\right)$

$$\left(L^H + \frac{w^F}{w^H} L^F\right) = L^H\left(\frac{\theta_1 + \theta_2}{\theta_1}\right)$$

Solving Free Trade Equilibrium

$$\left(L^H + \frac{w^F}{w^H} L^F\right) = L^H \left(\frac{\theta_1 + \theta_2}{\theta_1}\right)$$

Subtracting L^H from both sides

$$\frac{w^F}{w^H} L^F = L^H \left(\frac{\theta_1 + \theta_2}{\theta_1}\right) - L^H$$

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Simplifying

$$\frac{w^F}{w^H} L^F = L^H \left(\left(1 + \frac{\theta_2}{\theta_1}\right) - 1\right)$$

$$\frac{w^F}{w^H} L^F = L^H \left(\frac{\theta_2}{\theta_1}\right)$$

Solving Free Trade Equilibrium

$$\frac{w^F}{w^H} L^F = L^H \left(\frac{\theta_2}{\theta_1}\right)$$

Dividing by both sides gives **relative wages** (here Foreign wages relative to Home wages)

$$\frac{w^F}{w^H} = \frac{L^H}{L^F} \left(\frac{\theta_2}{\theta_1}\right)$$

Solving Free Trade Equilibrium

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Relative Wages = 1/(Relative Labor Supply) × Relative Consumption Share of Exported Good

- A country's relative **wages decrease** as it's relative **labor supply increases**
- A country's relative **wages increase** as **demand for its exported good increases**

Solving Free Trade Equilibrium

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Equilibrium relative prices are

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$$\frac{p_1}{w^H} = a_1^H$$

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- For good 2, we need to plug in relative wages in form of $w^F = w^H \frac{L^H}{L^F}\left(\frac{\theta_2}{\theta_1}\right)$, therefore

$$\frac{p_2}{\left(w^H \frac{L^H}{L^F}\left(\frac{\theta_2}{\theta_1}\right)\right)} = a_2^F \Rightarrow \frac{p_2}{w^H} = a_2^F \frac{L^H}{L^F}\left(\frac{\theta_2}{\theta_1}\right)$$

Solving Free Trade Equilibrium

We can then use prices to find **consumption allocations**

- Plugging in relative price for good 1 in Home country gives

$$c_1^H = \overbrace{(1/a_1^H)}^{w^H/p_1} L^H \left(\frac{\theta_1}{\theta_1 + \theta_2} \right)$$

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- Likewise for good 2,

$$c_2^H = \overbrace{\left((1/a_2^F)\frac{L^F}{L^H}\left(\frac{\theta_1}{\theta_2}\right)\right)}^{\frac{w^H}{p_2}=\frac{w^F}{p_2}\times\frac{w^H}{w^F}} \overbrace{L^H\left(\frac{\theta_2}{\theta_1+\theta_2}\right)}^{\text{Share of H income devoted to good 2}} = \overbrace{(1/a_2^F)}^{w^F/p_2} \overbrace{L^F\left(\frac{\theta_1}{\theta_1+\theta_2}\right)}^{\text{Share of F income devoted to good 1}}$$

The income share is just the theta term. Multiplying by $w^i L^i$ gives income allocated to good

Solving Free Trade Equilibrium

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- For Foreign country we follow the same process, and get

$$c_1^F = (1/a_1^H)L^H\left(\frac{\theta_2}{\theta_1+\theta_2}\right), \qquad c_2^F = (1/a_2^F)L^F\left(\frac{\theta_2}{\theta_1+\theta_2}\right)$$

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That gives us all allocations, as well as all relative prices, so we're done

Verifying Walras‘ Law

This time we used all the market clearing conditions. We didn’t use **balanced trade**

- We can verify it holds in our equilibrium (verifying Walras’ law is a great way to make sure your computer programs work)

Verifying Walras' Law

Recall, Balanced Trade equation under complete specialization is

$$\overbrace{p_2 c_2^H}^{\text{Value of Home Imports}} = \overbrace{p_1 c_1^F}^{\text{Value of Foreign Imports}}$$

Verifying Walras‘ Law

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Lets normalize $w^H = 1$. Plugging in our prices and consumption allocations gives

$$\overbrace{a_2^F \frac{L^H}{L^F}\left(\frac{\theta_2}{\theta_1}\right)}^{p_2} \overbrace{(1/a_2^F)L^F\left(\frac{\theta_1}{\theta_1+\theta_2}\right)}^{c_2^H} = \overbrace{a_1^H}^{p_1} \overbrace{(1/a_1^H)L^H\left(\frac{\theta_2}{\theta_1+\theta_2}\right)}^{c_1^F}$$

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Simplifying the above shows that it automatically holds at our equilibrium values

$$L^H\left(\frac{\theta_2}{\theta_1+\theta_2}\right) = L^H\left(\frac{\theta_2}{\theta_1+\theta_2}\right) \Rightarrow 1 = 1$$

Verifying Our Assumption of Complete Specialization

With our preferences and free trade, complete specialization always an equilibrium

- Worst case scenario: countries are identical ⇒ no gains/losses from trade. Multiple equilibria in this case; specialization is one of them.

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- Not true when we add tariffs and trade costs. Then countries may choose not to trade.

Numerical Example

- Let's verify specialization holds in our free trade equilibrium
- In general, depends on the **exogeneous parameters** of the model
- Set $L^H = L^F = 1$; $\theta_1 = \theta_2 = 1$. $a_1^H = a_2^F = 1$; $a_2^H = a_1^F = 2$.

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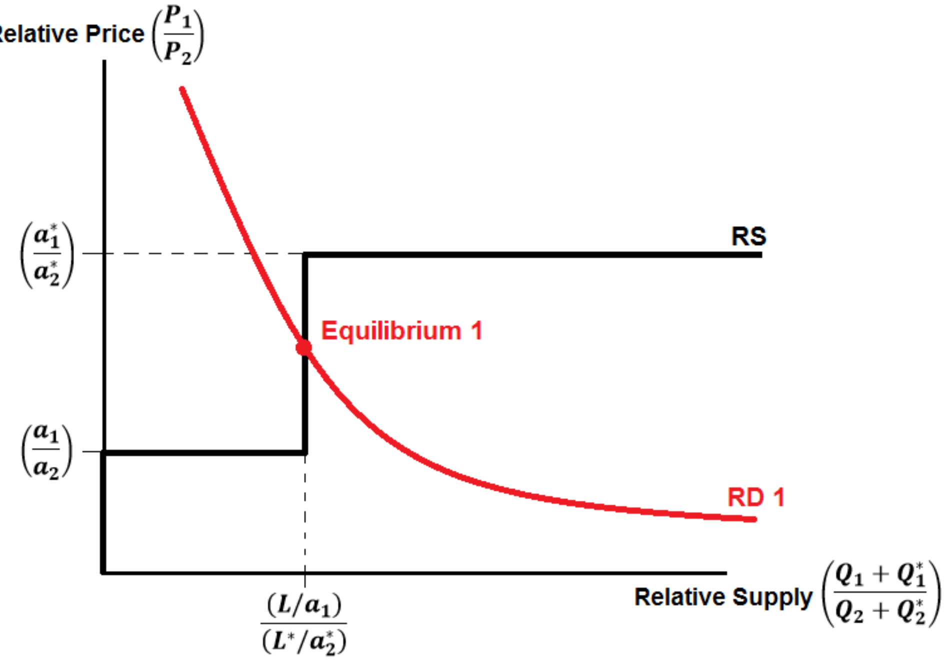
Relative unit input costs are

$$\frac{a_1^H}{a_2^H} = \frac{1}{2}; \frac{a_1^F}{a_2^F} = 2$$

Relative prices (now relative to each other) are

$$p_1 = a_1^H = 1 \text{ and } p_2 = a_2^F \frac{L^H}{L^F}\left(\frac{\theta_2}{\theta_1}\right) = 1 \Rightarrow \frac{p_1}{p_2} = 1$$

Finding Equilibrium using Relative Demand



$\frac{1}{2} < 1 < 2 \Rightarrow \left(\frac{a_1^H}{a_2^H}\right) < \left(\frac{p_1}{p_2}\right) < \left(\frac{a_1^F}{a_2^F}\right) \Rightarrow$ Home produces only good 1. Foreign produces only good 2.

(Graph: no star = Home, star = Foreign)