

## ECON 256: Final Exam Practice Math/Formula Questions SOLUTIONS

1. Given the following table, compute the mean value of  $X$  (i.e. compute  $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$ )

	Obs. 1	Obs. 2	Obs. 3
X	-4	13	27

$$\bar{x} = \frac{1}{3}(-4 + 13 + 27) = \frac{1}{3}(36) = 12$$

2. Calculate the covariance between  $X$  and  $Y$  for the below table, where

$$\text{Cov}(X, Y) \equiv \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})$$

Note that here  $\bar{x} = 7$  and  $\bar{y} = 4$ .

	Obs. 1	Obs. 2
X	13	1
Y	2	6

$$\begin{aligned} \text{Cov}(X, Y) &= \frac{1}{2-1} ((13-7)(2-4) + (1-7)(6-4)) \\ &= \frac{1}{1} ((6)(-2) + (-6)(2)) = (-12 + (-12)) = -24 \end{aligned}$$

3. The correlation coefficient between  $X$  and  $Y$  is defined as

$$r \equiv \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X)} \times \sqrt{\text{Var}(Y)}}$$

Suppose  $\text{Cov}(X, Y) = 3$ , while  $\text{Var}(X) = 4$  and  $\text{Var}(Y) = 9$ . What is  $r$  to 2 digits?

$$r = \frac{3}{\sqrt{4} \times \sqrt{9}} = \frac{3}{2 \times 3} = \frac{3}{6} = 0.5$$

4. Suppose we have the below regression relating hours spent procrastinating

$$\log(\text{Score}_i) = 95 - 3 \times \log(\text{Hours Procrastinating}_i) + \epsilon_i$$

If we increase hours spent procrastinating by 10%, by approximately what percent (percent, not percentage points) should we expect the score to rise/decline?

Important thing to note is that it's both  $\log(\text{score})$  and  $\log(\text{procrast.})$  in the regression.  
Therefore:

$$\% \text{ Change in Score} \approx -3 \times \% \text{ Change in Hours Procrastinating} = -3 \times 10 = -30$$

So would expect a 30% decline in score

5. Suppose we have the following table

Data Series	Observation 1	Observation 2	Observation 3	Observation 4	Observation 5
X	12	47	21	37	25
Y	-8	15.5	-39.5	41.5	-7.5

which gives us the estimated regression coefficients plugged into the formula before

$$y_i = -30 + 0.5x_i + \epsilon_i$$

What is the predicted value of  $Y$  when  $X = 47$ ? What is the error for observation 2?

$$\hat{Y} = -30 + 0.5 \times 47 + 0 = -30 + 23.5 = -6.5$$

What is the error for observation 2? Two ways, both give same answer:

$$\text{Method 1: } \epsilon_i = y_i - (-30 + 0.5x_i) = y_i + 30 - 0.5x_i \Rightarrow$$

$$\epsilon_2 = 15.5 + 30 - 0.5 * 47 = 45.5 - 23.5 = 22$$

$$\text{Method 2: } \epsilon_i = y_i - \hat{Y}, \text{ where } \hat{Y} \text{ is the predicted value of } Y \text{ when } X = x_i. \Rightarrow$$

$$\epsilon_2 = 15.5 - (-6.5) = 15.5 + 6.5 = 22$$

6. For the above regression, how much would we expect  $Y$  to change between  $X = 12$  and  $X = 47$ ? How much did it actually change by?

$$\text{Predicted Change} = \Delta \hat{Y} = \beta_X(X' - X) = 0.5 \times (47 - 12) = 0.5 \times 35 = 17.5$$

$$\text{Actual Change} = y_2 - y_1 = 15.5 - (-8) = 15.5 + 8 = 23.5$$

7. In the above regression we have an estimated slope coefficient  $\beta_X = 0.5$ . The standard error of the estimate is approximately 0.97. What is the 95% confidence interval for  $\beta_X$ ? Is  $\beta_X$  statistically significant?

95 % confidence interval for  $\beta_X$  is approximately

$$\begin{aligned} & [\beta_X - 2 \times SE, \beta_X + 2 \times SE] \\ & = [0.5 - (2 \times 0.97), 0.5 + (2 \times 0.97)] \\ & = [0.5 - 1.94, 0.5 + 1.94] \\ & = [-1.44, 2.44] \end{aligned}$$

The estimate is **NOT** statistically significant. This is because zero falls within the confidence interval (equivalently, the confidence interval contains both a positive and a negative number; indicating we are less than 95% sure that the number is positive)

8. Suppose we have the Cobb-Douglas production function

$$Y = K^\alpha L^{1-\alpha}$$

where  $\alpha = 1/3$ . What should the optimal share of spending on labor be if there is no misallocation? What should the ratio of spending on capital to labor be?

If there is no misallocation, then should have spending on labor equal to the factor intensity. That means

$$\text{Share of Spending on Labor} = \frac{wL}{rK + wL} = 1 - \alpha = 1 - \frac{1}{3} = \frac{2}{3}$$

Equivalently, Share of Spending on Capital =  $\frac{rK}{rK+wL} = \alpha = 1/3$ . Therefore

$$\frac{\text{Spending on Capital}}{\text{Spending on Labor}} = \frac{rK}{wL} = \frac{rK/(rK + wL)}{wL/(rK + wL)} = \frac{\alpha}{1 - \alpha} = \frac{1/3}{2/3} = \frac{1}{2}$$

9. Suppose we have the following table

Category	Millions of People
Number of 12-19 Year Olds Enrolled in Secondary School	5
Number of <b>Non</b> -12-19 Year Olds Enrolled in Secondary School	2
Population of 12-19 Year Olds	11
Total Population	30

What is the Gross Secondary School Enrollment Rate in percent?

$$\text{Gross Enrollment Rate} = \frac{\text{\#People Enrolled in Secondary School}}{\text{Population of 12 - 19 Year Olds}} = \frac{5 + 2}{11} = 0.63 = \mathbf{63\%}$$

10. For the above table, what is the Net Secondary School Enrollment Rate in percent?

$$\text{Net Enrollment Rate} = \frac{\text{\#12-19 Year Olds in Secondary School}}{\text{Population of 12 - 19 Year Olds}} = \frac{5}{11} = 0.45 = \mathbf{45\%}$$

11. Suppose a bank pays depositors 2%. How high of an interest rate would a bank have to charge on a loan to break even if there is only a 30% chance of repayment? Recall formula for break even interest rate is given by:

$$i = \frac{1 + r}{p} - 1$$

First note we have to convert the numbers from percents to rates. So  $r = 0.02$  (2% deposit rate) and  $p = 0.3$  (30% repayment rate). Therefore break even interest rate is:

$$i = \frac{1 + 0.02}{0.3} - 1 = \frac{1.02}{0.3} - 1 = 3.5 - 1 = 2.5$$

which, converting back to a percent, is a **250% interest rate**. I.e. the interest rate must be 125 times larger than the deposit rate for the bank to break even.

12. Suppose a borrower has **zero liability** if a project fails. Suppose the borrower invests in a risky project where there is a **30% chance of failure** where the borrower receives nothing. There is also a **70% chance of success** where the borrower gets **5 times their investment**.

Suppose the interest rate is **50%**. What is the expected return for a borrower who borrows **\$100** to invest in a risky project? What is the expected return for the bank giving the loan?

Would both the bank and borrower want this loan to happen with the given interest rate?

For Borrower: If **project succeeds (70% chance)**, project pays 500. **100** goes back to original loan, **50** goes to interest; leaving borrower with **350** profit.

$$\text{Return for Borrower} = 0.3 \times 0 + 0.7 \times 350 = 245$$

For Consumer: If **project succeeds (70% chance)**, bank gains 50 (original 100 doesn't count as a gain). If project fails, bank loses initial loan and receives no interest, so loses 100.

$$\text{Return for Bank} = 0.3 \times -100 + 0.7 \times 50 = -30 + 35 = 5$$

**Yes, the loan would happen**, since both the Borrower and Bank have a positive expected return.