ECON 256 – Worksheet 3
Spring 2017

Problem 1 For this exercise, we will a production function of the form

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

Q1.1) Suppose that $\alpha = 0.3$ and $A_t = 10$. Use the formula to fill in output for the following table

Capital Input	Labor Input	Output
15	15	$10 \times (15)^{0.3} \times (15)^{0.7} = 150$
30	15	$10 \times (30)^{0.3} \times (15)^{0.7} = 185$
15	30	$10 \times (15)^{0.3} \times (30)^{0.7} = 244$
30	30	$10 \times (30)^{0.3} \times (30)^{0.7} = 300$

Q1.2 Does output more than double, double, or less than double when you double capital input and hold labor input fixed? What about if you double both inputs? What if you hold inputs fixed, but double TFP?

Problem 2

The dynamics of the Solow growth model are explained by the following two equations:

- Accumulation of Capital per Worker

$$\frac{K_{t+1}}{L_{t+1}} = \frac{L_t}{L_{t+1}} \left[(1-\delta)\frac{K_t}{L_t} + s\frac{Y_t}{L_t} \right], \quad (1)$$

- Output per Worker

$$\frac{Y_t}{L_t} = A_t \left(\frac{K_t}{L_t} \right)^{\alpha}, \quad (2)$$

Suppose that $A_t = 10$, $L_{t+1} = L_t$ (there is no population growth), and $\alpha = 0.3$. Further suppose the savings rate is $s = 0.05$ and the depreciation rate is $\delta = 0.10$. Then the equations become:

- Accumulation of Capital per Worker

$$\frac{K_{t+1}}{L_{t+1}} = (1-0.1)\frac{K_t}{L_t} + 0.05\frac{Y_t}{L_t}, \qquad (1)$$

- Output per Worker

$$\frac{Y_t}{L_t} = 10 \left(\frac{K_t}{L_t} \right)^{0.3}, \quad (2)$$

Suppose we start with an initial Capital-Labor ratio of $K_0/L_0 = 1$. Using the above two equations fill out the following table (Hint: Find Y_0/L_0 then use it with K_0/L_0 to get K_1/L_1 and so on).

Time	0	1	2
K/L	$\mathbf{1}$	$0.9 \times \mathbf{1} + 0.05 \times \mathbf{10} = \mathbf{1.4}$	$0.9 \times \mathbf{1.4} + 0.05 \times \mathbf{11.1} = \mathbf{1.81}$
Y/L	$10 \times (\mathbf{1})^{0.3} = \mathbf{10}$	$10 \times (\mathbf{1.4})^{0.3} = \mathbf{11.1}$	$10 \times (\mathbf{1.81})^{0.3} = \mathbf{11.95}$

**note, answers may differ slightly due to rounding errors. The 0.9 is coming from $(1-0.1)$

Problem 3

Steady state Output per Worker in the Solow growth model is given by the equation

$$\frac{Y}{L} = A^{\frac{1}{1-\alpha}}\left(\frac{s}{\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

And steady state capital per worker is given by

$$\frac{K}{L} = \left(A\frac{s}{\delta}\right)^{\frac{1}{1-\alpha}}$$

Where A is productivity, or TFP, s is the savings rate, δ is the depreciation rate, and α is Capital's ideal share of output.

3.a) Based on the formula above say what happens to when each of the following occurs

Change	Effect on SS Capital per Worker	Effect on SS Output per Worker
$A\uparrow$	increases	increases
$s\downarrow$	decreases	decreases
$\delta\downarrow$	increases	increases

3.b) Let $A = 1$, $\alpha = 1/2$, $s = 0.2$, and $\delta = 0.05$. Find the steady state values of Y/L and K/L [Hint: You shouldn't need a calculator].

$$\frac{Y}{L} = (1)^{\frac{1}{1-\frac{1}{2}}}\left(\frac{0.2}{0.05}\right)^{\frac{\left(\frac{1}{2}\right)}{1-\frac{1}{2}}} = (1)^{\frac{1}{1/2}}(4)^{\frac{(1/2)}{(1/2)}} = 1 \times (4)^1 = 4$$

$$\frac{K}{L} = \left(1 \times \frac{0.2}{0.05}\right)^{\frac{1}{1-\frac{1}{2}}} = (4)^{\frac{1}{1/2}} = (4)^2 = 16$$

3.c) Suppose that $\alpha = 0$, so that capital is no longer used in the production of output. What is the new steady state value of capital per worker? Do you think this makes sense?

$$\frac{K}{L} = \left(1 \times \frac{0.2}{0.05}\right)^{\frac{1}{1-0}} = (4)^1 = 4$$

Why should consumers continue to invest in capital if it is not used in production? What do you think the steady state value of K/L should be if $\alpha = 0$, ignoring the above equations?

They shouldn't. It's a shortcoming of the model. The savings rate should be determined endogenously rather than being given exogenously as a fixed fraction of income. When we do growth accounting, we will use a framework that allow the savings rate to change when the economy changes.

With $\alpha = 0$, the SS value of K/L should be zero if households are able to choose how much they invest.