**ECON 442: Problem Set 1** Professor Jack Rossbach

Due Date: Tuesday, Feb 28 at 5:00PM Spring 2017

**Problem Set 1**

Complete the questions for each problem. Answers must be typed and uploaded to canvas as either a word document or pdf. You are encouraged to discuss the problems with each other, however, everybody needs to submit their own assignment and type up their own answers.

**Problem 1**: **Transfers and trade with non-traded goods.**

Suppose we have two countries and three goods.

- Good zero is non-traded, and produced by both countries with labor cost $a_0^H = a_0^F = 1$
- Good 1 is tradable, and can be produced only by country H. Country H has labor cost $a_1^H$ for good 1.
- Good 2 is tradable, and can be produced only by country F. Country F has labor cost $a_2^F$ for good 2.

Marginal labor costs are constant (effectively, $a_1^F = a_2^H = \infty$) and Labor supply in country $i$ is $L^i$

Preferences are given by

$$U^i(c_0^i, c_1^i, c_2^i) = \theta \log c_0^i + \left(\frac{1-\theta}{2}\right)(\log c_1^i + \log c_2^i)$$

Suppose Country $F$ pays a flat transfer to country $H$ equal to $T$ ($T \geq 0$).

**Questions**

**1.i)** Write down and give the solution for the consumer's problem (you can do it for country $i$ where $i$ can be either $H$ or $F$; or just do it for one country so you don't have to write everything twice)

**1.ii)** Write down the firm's problem. What does the firm's problem imply about wages and prices?

**1.iii)** Write down the market clearing constraints. What does a balanced current account mean?

**i.iv)** What do preferences imply about the utility of consumers if they cannot consume one of the goods? What do we know about what the pattern of specialization will be in the free trade equilibrium?

**i.v)** Define an equilibrium for this economy. (You do not need to re-type the equations)

**i.vi)** Solve for the equilibrium of this economy using the code that we write together in class. Don't forget to normalize one of the prices or wages.

**i.vii)** What happens to relative wages ($w^F/w^H$) and relative prices ($p_2/p_1$) as the transfer changes. How does this compare to an economy where there is no non-traded good?

Does what happens to the Terms of Trade affect how much the transfer hurts the Foreign country? In what way?

**Problem 2**

For this problem we are going to combine Comparative Advantage with another reason to trade: Taste for Variety.

There are two countries, $i, j = H, F$, and two types of goods, $m = 1,2$. For each good, each country produces their own variety of that good. Preferences in country $i$ are given by

$$U^i\left(c_{1H}^i, c_{2H}^i, c_{1F}^i, c_{2F}^i\right) = \theta_1 \log\left(\left(c_{1H}^i\right)^\rho + \left(c_{1F}^i\right)^\rho\right) + \theta_2 \log\left(\left(c_{2H}^i\right)^\rho + \left(c_{2F}^i\right)^\rho\right)$$

Where $c_{1H}^i$ is the consumption of H's variety of good 1 in country $i$. Similarly for the other goods, $c_{mj}^i$ is the amount of $j$'s variety of good 1 consumed in country $i$. We require $\rho \in (0,1)$; $\theta_1, \theta_2 > 0$.

These preferences imply that consumers want to consume some of each variety of each good. They are referred to as CES (Constant Elasticity of Substitution) preferences, since the elasticity of substitution between varieties for each good is constant and equal to $\frac{1}{1-\rho}$.

The budget constraint for consumers in country $i$ is standard:

$$\sum_{m=1}^{2}\sum_{j=1}^{2} p_{mj}^i c_{mj}^i = w^i L^i$$

Where $p_{mj}^i$ is the price of variety $j$ of good $m$ consumed in country $i$. Prices may differ across countries, as there are iceberg trade costs to ship goods. This means that to export 1 unit of a good, it is necessary to ship $\tau \geq 1$ units ($\tau = 1$ is frictionless trade).

Firms are perfectly competitive and internalize trade costs into their production. Firms located in country $j$ for good $m$ can produce output for country $i$ according to the production function:

$$y_{mj}^i = \frac{1}{\tau_j^i a_{mj}} l_{mj}^i$$

Where $\tau_j^j = 1$ (no trade costs to serve own market) and $\tau_j^i = \tau$ if $i \neq j$. $a_{mj}$ is the unit labor cost for firms to produce one unit of good $m$ in country $j$ for the domestic market.

Due to trade costs, we require markets clear individually for each market. That means there are 8 goods market clearing conditions:

$$c_{mj}^i = y_{mj}^i, \qquad m = 1,2; i, j = 1,2$$

Labor market clearing for country is given by

$$\sum_{m=1}^{2}\sum_{i=1}^{2} l_{mj}^i = L^j, \qquad j = 1,2$$

Use this model to answer the questions on the following page.

**Questions**

**2.i)** What are the exogenous parameters for the model? What are the endogenous parameters for the model? (You can leave out subscripts and superscripts for this question)

**2.ii)** The solution to the consumer problem is

$$c_{mj}^{i} = \frac{\left(\frac{\theta_m}{\theta_1 + \theta_2}\right) w^i L^i}{\left(p_{mj}^{i}\right)^{\frac{1}{1-\rho}} \left(P_m^i\right)^{-\left(\frac{\rho}{1-\rho}\right)}}, \qquad m = 1,2; i,j = 1,2$$

Where $P_m^i$ is an aggregated price index for good $m$ for consumers in country $i$

$$P_m^i \equiv \left(\left(p_{mH}^{i}\right)^{-\left(\frac{\rho}{1-\rho}\right)} + \left(p_{mF}^{i}\right)^{-\left(\frac{\rho}{1-\rho}\right)}\right)^{-\left(\frac{1-\rho}{\rho}\right)}, \qquad m = 1,2; i = 1,2$$

Firms are perfectly competitive, so the solution to the firms problem is given by

$$p_{mj}^{i} = \tau_j^i a_{mj} w^j, \qquad m = 1,2; i,j = 1,2$$

**Q:** How many equilibrium equations are there for the consumers' problems? How many equilibrium equations are there for the firms' problems? How many total equilibrium equations?

**2.iii)** On my website you will find notes showing how to solve the model algebraically in the case where countries are symmetric.

Assume $\theta_1 = \theta_2 = 1$; $\tau = 1$; $L^H = L^F = 10$; $a_{1H} = a_{2H} = a_{1F} = a_{2F} = 1$; $\rho = 0.5$. Plug these values into the algebraic solutions to find equilibrium prices and allocations.

**2.iv)** Solve the model on the computer for the same parameter values as above. Make sure that it is easy to update the parameter values in your code, since we will be changing them. Normalize $w^H = 1$ and make sure to remember Walras' law. Verify that your model gives the same solution as you got above. [Note: it can be useful to plug in market clearing conditions by hand to reduce the number of equilibrium variables and equations you need to solve for on the computer].

Now suppose that there are iceberg costs of 10%, so that $\tau = 1.1$. Report the new equilibrium.

**2.v)** Suppose that Home is a large country so that $L^H = 100$. (change $L^H$ for the guess in the code also)

Compute the real income per capita index, which is defined as

$$\exp\left[U^i\left(\frac{c_{1H}^i}{L^i}, \frac{c_{2H}^i}{L^i}, \frac{c_{1F}^i}{L^i}, \frac{c_{2F}^i}{L^i}\right)\right] \equiv \left(\left(\frac{c_{1H}^i}{L^i}\right)^{\rho} + \left(\frac{c_{1F}^i}{L^i}\right)^{\rho}\right)^{\theta_1} \left(\left(\frac{c_{2H}^i}{L^i}\right)^{\rho} + \left(\frac{c_{2F}^i}{L^i}\right)^{\rho}\right)^{\theta_2}$$

Report the value of the index (the RHS of above formula) for each country when we move from a world with iceberg costs ($\tau = 1.1$) to a world with frictionless trade ($\tau = 1$). What do our results say about whether large or small countries tend to benefit more from free trade?

**2.vi)** Let $L^H = 10$ again (and $\tau = 1$). Suppose Home excels at producing good 1, so that $a_{1H} = 1/2$.

What happens to labor allocations for each country for each good compared to 2.iv? Why?

**Problem 3. Revealed Comparative Advantage**

Consider the following index from Balassa (1965)

$$RCA_i(z) = \left(\frac{X_i(z)}{X_i(total)}\right) / \left(\frac{X_{world}(z)}{X_{world}(total)}\right)$$

Where $X_i(z)$ is country $i$'s exports of good $z$, $X_i(total)$ is country $i$'s total exports, $X_{world}(z)$ is the World's exports of good $z$, and $X_{world}(total)$ is the World's total exports.

If $RCA_i(z) > 1$, we say that country $i$ has a revealed comparative advantage in good $z$.

**Questions**

**3.i)** Register an account on http://wits.worldbank.org/

Download data on Pakistan's Exports and "All Countries" Exports to the World in 1995 and 2015 at the 4-digit SITC Rev 2 level.

Using the Balassa formula, compute the RCA Index for each product that Pakistan exports in 1995. Report the 5 highest RCA Index values and what products they belong to.

**3.ii)** Report the fraction of Pakistan's exports, in terms of the value of trade flows, in 2015 that were in products that were not exported by Pakistan in 1995.

For the remaining exercises, exclude products that were not exported by Pakistan in 1995.

**3.iii)** Compute the percentage change in Pakistan's exports between 1995 and 2015 for each product, deflated by the percent change of Pakistan's GDP between 1995 and 2015 according to the following formula:

$$\%\text{Change}(z) = 100 \times \left(\frac{X_{2015}(z)/GDP_{2015}}{X_{1995}(z)/GDP_{1995}} - 1\right)$$

Use data from the World Development Indicators for Pakistan's GDP in 1995 and 2015. The series you should use is "GDP (current US$)".

What is the total value of all exports in 1995 and 2015 for Pakistan, as a fraction of GDP? What is the correlation between the RCA Index and the Percentage Change in Exports?

**3.iv)** Report the median growth (percent change) of exports with a RCA Index less than 1, and the median growth of exports with a RCA Index greater than 1.

What was the total growth for the exports of all exports of products with an RCA Index less than 1? The total growth for all exports of products with an RCA Index greater than 1? Which set of products experienced greater growth?

**3.v)** Do the results in 3.iv) surprise you or make sense in the context of the Ricardian model? Why?

Skim this paper: http://www.nber.org/papers/w17969.pdf

Can you think of any reasons why the Balassa RCA Index may not be a good measure of a country's true Comparative Advantage?