ECON 442: Quantitative Trade Models **Professor Jack Rossbach**

Description of Multidimensional Ricardian (Eaton-Kortum) Model Equations for R

The solution to the equilibrium is characterized by the following equations:

$$w_i L_i = \sum_{n=1}^{3} \pi_{ni} w_n L_n , \qquad i = 1,2,3$$

$$\pi_{ni} = T_i \left(\frac{\gamma d_{ni} w_i}{p_n}\right)^{-\theta} , \qquad i, n = 1,2,3$$

$$p_n = \gamma \left[\sum_{i=1}^{3} T_i (d_{ni} w_i)^{-\theta}\right]^{-\left(\frac{1}{\theta}\right)} , \qquad n = 1,2,3$$

We can listing all the equations described by the above by plugging in the different values of i and n. The equations are in the table on the following page. Below are a few notes.

- For the equations, I expand the sums and plug in $L_1 = L_2 = L_3 = 1$.
- Note that if you plug p_n into π_{ni} the γ terms cancel. Therefore I leave γ out of both sets of equations since it won't affect our solution for π_{ni}
- Technically, leaving the γ term out means that p and therefore w/p will be incorrect, but we are only interested in changes in welfare, not the units of welfare itself. When we do changes in welfare the γ would cancel again since it's a constant, meaning our welfare changes are correct.
- You can verify the above bullet if you want, we need to use the parameter, $\sigma = 2$, in which case we can compute gamma in R with the code:

  ```
  gamma <- gamma((theta+1-sigma)/theta)^(1/(1-sigma))
  ```

  The gamma() on the RHS (I made it red) is actually the gamma function, the gamma on the LHS is a variable name for our parameter γ. That's why you always need to put asterisks to multiply in R, because otherwise R thinks you are trying to plug inputs into a function rather than multiply variables.
- σ is only used in the γ term, so I leave both of those variables out of the tables below. The reason these terms disappear is because we're doing a special case of the model in which production does not use intermediate goods. If we allow for intermediate goods, then we need those terms.

Expanded Table of Equilibrium Equations for R

R Name	Equilibrium Equation	Values plugged in for i and n
#eqns[0]	$w_1 = (\pi_{11}w_1 + \pi_{21}w_2 + \pi_{31}w_3)$	$i = 1$
eqns[1]	$w_2 = (\pi_{12}w_1 + \pi_{22}w_2 + \pi_{32}w_3)$	$i = 2$
eqns[2]	$w_3 = (\pi_{13}w_1 + \pi_{23}w_2 + \pi_{33}w_3)$	$i = 3$
eqns[3]	$\pi_{11} = T_1\left(\frac{d_{11}w_1}{p_1}\right)^{-\theta}$	$i = 1, n = 1$
eqns[4]	$\pi_{21} = T_1\left(\frac{d_{21}w_1}{p_2}\right)^{-\theta}$	$i = 1, n = 2$
eqns[5]	$\pi_{31} = T_1\left(\frac{d_{31}w_1}{p_3}\right)^{-\theta}$	$i = 1, n = 3$
eqns[6]	$\pi_{12} = T_2\left(\frac{d_{12}w_2}{p_1}\right)^{-\theta}$	$i = 2, n = 1$
eqns[7]	$\pi_{22} = T_2\left(\frac{d_{22}w_2}{p_2}\right)^{-\theta}$	$i = 2, n = 2$
eqns[8]	$\pi_{32} = T_2\left(\frac{d_{32}w_2}{p_3}\right)^{-\theta}$	$i = 2, n = 3$
eqns[9]	$\pi_{13} = T_3\left(\frac{d_{13}w_3}{p_1}\right)^{-\theta}$	$i = 3, n = 1$
eqns[10]	$\pi_{23} = T_3\left(\frac{d_{23}w_3}{p_2}\right)^{-\theta}$	$i = 3, n = 2$
eqns[11]	$\pi_{33} = T_3\left(\frac{d_{33}w_3}{p_3}\right)^{-\theta}$	$i = 3, n = 3$
eqns[12] & Pindex[1]	$p_1 = \left(T_1(d_{11}w_1)^{-\theta} + T_2(d_{12}w_2)^{-\theta} + T_3(d_{13}w_3)^{-\theta}\right)^{-\left(\frac{1}{\theta}\right)}$	$n = 1$
eqns[13] & Pindex[2]	$p_2 = \left(T_1(d_{21}w_1)^{-\theta} + T_2(d_{22}w_2)^{-\theta} + T_3(d_{23}w_3)^{-\theta}\right)^{-\left(\frac{1}{\theta}\right)}$	$n = 2$
eqns[14] & **Pindex[3]**	$p_3 = \left(T_1(d_{31}w_1)^{-\theta} + T_2(d_{32}w_2)^{-\theta} + T_3(d_{33}w_3)^{-\theta}\right)^{-\left(\frac{1}{\theta}\right)}$	$n = 3$

Table Discussion

- eqns[0] holds due to Walras' law, so we comment it out by putting a # in front of it in R and we normalize $w_1 = 1$. We could normalize a different wage instead if we wanted.
- **Bolded** equations are ones that are left for you to enter in the code template.
- When we enter the code, we need to use names that R can recognize for all the parameters and variables. Below are the names I gave each thing in R, in both condensed and expanded tables.
- Note that when we enter the code, we want to set the equations equal to zero. For that reason we will be subtracting the RHS of the above equations from the LHS when writing the code in R (e.g. for eqns[0] we write $w_1 - \pi_{11}w_1 - \pi_{21}w_2 - \pi_{31}w_3 = 0$, although we leave off the $= 0$ at the end)

Condensed Tables for Model Variables and Parameters

R Name	Equilibrium Variable	Description
w[i]	w_i	Wages in country i
pi[n,i]	π_{ni}	Expenditure Share in country i of imports from country n
p[n] & Pindex[n]	p_n	Consumer Price Index (CPI) in country n

R Name	Model Parameter	Description
theta	θ	Variation around Average Productivity in Each Country
T[i]	T_i	Average Productivity in Country i
d[n,i]	d_{ni}	Trade costs for country i to import from country n

Expanded Tables for Model Variables and Parameters

R Name	Equilibrium Variable	Description
w[1]	w_1	Wages in country 1
w[2]	w_2	Wages in country 2
w[3]	w_3	Wages in country 3
pi[1,1]	π_{11}	Exp. Share in country 1 of imports from country 1
pi[2,1]	π_{21}	Exp. Share in country 1 of imports from country 2
pi[3,1]	π_{31}	Exp. Share in country 1 of imports from country 3
pi[1,2]	π_{12}	Exp. Share in country 2 of imports from country 1
pi[2,2]	π_{22}	Exp. Share in country 2 of imports from country 2
pi[3,2]	π_{32}	Exp. Share in country 2 of imports from country 3
pi[1,3]	π_{13}	Exp. Share in country 3 of imports from country 1
pi[2,3]	π_{23}	Exp. Share in country 3 of imports from country 2
pi[3,3]	π_{33}	Exp. Share in country 3 of imports from country 3
p[1] & Pindex[1]	p_1	Consumer Price Index (CPI) in country 1
p[2] & Pindex[2]	p_2	Consumer Price Index (CPI) in country 2
p[3] & Pindex[3]	p_3	Consumer Price Index (CPI) in country 3

R Name	Model Parameter	Description
theta	θ	Variation around Average Productivity in Each Country
T[1]	T_1	Average Productivity in Country 1
T[2]	T_2	Average Productivity in Country 2
T[3]	T_3	Average Productivity in Country 3
d[1,1]	d_{11}	Trade costs for country 1 to import from country 1
d[2,1]	d_{21}	Trade costs for country 1 to import from country 2
d[3,1]	d_{31}	Trade costs for country 1 to import from country 3
d[1,2]	d_{12}	Trade costs for country 2 to import from country 1
d[2,2]	d_{22}	Trade costs for country 2 to import from country 2
d[3,2]	d_{32}	Trade costs for country 2 to import from country 3
d[1,3]	d_{13}	Trade costs for country 3 to import from country 1
d[2,3]	d_{23}	Trade costs for country 3 to import from country 2
d[3,3]	d_{33}	Trade costs for country 3 to import from country 3