ECO 442: Quantitative Trade Models

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Why Do Countries Trade?

Ricardian: Countries have different technologies for producing goods. Comparative advantage.

Heckscher-Ohlin: Countries have same technology for producing goods, but different factor endowments which are used in the production of the goods.

Armington: Countries have different goods, and consumers like to consume foreign goods.

Monopolistic Competition: Countries have firms which produce differentiated varieties of a good. Consumers like to consumer different varieties.

Increasing Returns: Cheaper to produce a good all in a single place, so countries should specialize and trade.

Formalizing the 2x2 Ricardian Model of Trade

What stays the same?

- Two goods, two countries.
- One factor of production, labor, in fixed supply
- Countries differ in their relative unit labor costs for producing each good

What was missing?

- Didn't specify preferences (Demand for each good)
- Previously illustrated intuition graphically; now derive equilibrium algebraically
- Seperate consumer problem from producer problem

Formalizing the 2x2 Ricardian Model of Trade

Why Algebra?

- Allows us to be precise.

Why Precise?

- People like answers with actual numbers
- Being precise in our answers allows us to evaluate our models
- Carefully specifying our models helps us understand how they work
- Understanding our models allows us to develop better ones

How We Solve Competitive Models

1. Specify agents: Consumers, Producers, occasionally Government
2. Set up optimization problems for each agent
3. Specify global constraints: Market Clearing
4. Solve for Competitive Equilibrium

What is the Competitive Equilibrium?

- Prices and allocations such that all agents are happy with their choices, given the choices of other agents

Consumer Preferences

- Large literature on preferences, rational choice theory, etc. Going to ignore all that stuff and skip to assuming a functional form utility function.
- For consumers: Higher value of utility function = better. Same value = indifferent.

Our utility functions will satisfy standard properties

- Monotone: More of everything is better than less of everything
- Transitive: A is better than B and B is better than C ⇒ A is better than C
- Complete: Can compute value of utility function for all possible allocations

Example of Utility function

Let there be two goods: c_1 is consumption of good 1, c_2 is consumption of good 2.

- Cobb-Douglas Utility Function:

$$U(c_1, c_2) = (c_1)^{\theta_1}(c_2)^{\theta_2}$$

Suppose $\theta_1 = \theta_2 = 0.5$. Would consumer prefer $(c_1, c_2) = (1,2)$ or $(1.5,1.5)$?

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Suppose $\theta_1 = \theta_2 = 0.5$. Would consumer prefer $(c_1, c_2) = (1,2)$ or $(1.5,1.5)$?

$$U(1,2) = (1)^{0.5}(2)^{0.5} \approx 1.41$$

$$U(1,2) = (1.5)^{0.5}(1.5)^{0.5} \approx 1.5$$

$U(1.5,1.5) > U(1,2)$ therefore consumer prefers second allocation.

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Important: Utility doesn't have natural units. Only relative utility matters.

- Transformations that preserve ordering are considered equivalent utility functions.

- Common order preserving transofrmations: Addition, Multiplication, Powers, Logairthms

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Example Transformation: Take logarithm

$$\widetilde{U}(c_1, c_2) = \theta_1 \log c_1 + \theta_2 \log c_2$$

$\widetilde{U}(c_1, c_2)$ is the same utility function as $U(c_1, c_2)$ [Note $\log 0 = -\infty$, always consume some of both]

Consumer Problem

Consumer problem will be to maximize utility function, subject to budget constraint

Budget Constraint

- Without a budget constraint, consumers would want an infinite amount of everything
- Budget constraint enforces that consumer expenditures are less than consumer income
- Typically no borrowing or saving in this class (we focus mainly on static models)

Consumption Expenditures: Sum of expenditures (= price * quantity) across all goods.

Income Sources: Labor income (wages * labor supplied).
Other potential sources: rental rates from capital, profits from firms, taxes from government

Consumer Problem

Given prices $\{p_1, p_2, w\}$, consumers choose consumption $\{c_1, c_2\}$ to **Maximize Utility**

$$\max_{\{c_1, c_2\}} U(c_1, c_2)$$

Subject to **budget constraint**

$$p_1 c_1 + p_2 c_2 \leq wL$$

And subject to non-negativity constraints

$$c_1 \geq 0; c_2 \geq 0$$

- Will suppress non-negativity constraints in the future for notational simplicity

Consumer Problem

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Subject to **budget constraint**

$$p_1 c_1 + p_2 c_2 \leq wL$$

Note 1: When solving the problem we will specify a functional form for the utility function

Note 2: Consumers do not choose how to allocate their labor between good 1 and 2, only how much labor to supply. The firms that hire them determine that (wages must be equal for both).

Firm Problem

For basic Ricardian model we assume firms are **perfectly competitive**.

- This means there are no profits and firms have no market power (they take prices as given)

All firms within a country assumed to have same production technology for a given good

- Typically assume **constant returns to scale (CRS)**: double inputs ⇒ double outputs
- Production technologies vary across products, not firms
- For now, assume single product firms

Production Functions

- Production functions specify how inputs are transformed into outputs.

- Example of production function with capital would be Cobb-Douglas:

$$Y = f(K, L) = AK^{\theta}L^{1-\theta}$$

Where Y is output, $f(K, L)$ is the production function, A is TFP, K is capital, L is labor, $\theta \in [0,1]$

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Where Y is output, $f(K, L)$ is the production function, A is TFP, K is capital, L is labor, $\theta \in [0,1]$

- For now, we will have only labor. Special case of above with $\theta = 0$.

$$y_m = z_m L_m$$

Where y_m is output of good m, l_m is labor used to produce good m, and z_m is labor productivity

Reminder: $a_m = 1/z_m$ (unit labor costs equal reciprical of productivity)

Important: Output has natural units. Do not take transformations of production functions!

Firm Optimization Problem

Assume firms **Maximize Profit**, where profit equal revenues minus costs

Firm that produce good m solve:

$$\max_{\{y_m, l_m\}} p_m y_m - w_m l_m$$

Subject to their production function:

$$y_m = \frac{1}{a_m} l_m$$

(No profits in equilibrium, but irrelevant to firm problem right now)

Market Clearing Conditions

The last part of the problem is to specify market clearing conditions

Labor Market Clears: labor demand = labor supply in each country

$$l_1^H + l_2^H = L^H$$

$$l_1^F + l_2^F = L^F$$

Goods Market Clears: output of each good = consumption of each good

Important: This condition changes depending on Trade vs Autarky

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Autarky: Goods market clearing for Home is to consume what is produced at Home

$$c_1^H = y_1^H$$

$$c_2^H = y_2^H$$

(In Autarky, everything that happens in Foreign is irrelevant to equilibrium in Home)

Market Clearing Conditions

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Labor Market Clears: labor demand = labor supply in each country

$$l_1^H + l_2^H = L^H$$

$$l_1^F + l_2^F = L^F$$

Goods Market Clears: output of each good = consumption of each good

Trade: Countries don't need to consume what they produce

$$c_1^H + c_1^F = y_1^H + y_1^F$$

$$c_2^H + c_2^F = y_2^H + y_2^F$$

Equilibrium Definition

Equilibrium is prices $\{p_1, p_2\}$, wages, $\{w_H, w_F\}$ and allocations $\left\{c_1^i, c_2^i; y_1^i, y_2^i\right\}_{i \in \{H,F\}}$ such that

1. Consumers maximize utility

2. Firms maximize profits

3. Markets clear

Exogenous vs Endogenous Variables

When working with models, keep in mind what is **Exogenous** vs **Endogenous**

Exogenous variables are parameters that are determined outside of the model

- In our model: productivity parameters, preference parameters, total labor supply

Endogenous variables are parameters that are determined by the model in equilibrium

- In our model: wages and prices, labor and consumption allocations across goods
- Equilibrium outcomes for endogenous variables are affected by exogenous parameters. The opposite is not true.

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Things that are exogenous in one model are often endogenous in another. Exogenous also does not mean arbitrary, we can estimate exogenous parameters using data.

Solving For the Equilibrium: Consumer Problem

We start by solving the consumer maximization problem. For now, suppress country.

Assume utility is a logged Cobb-Douglas utility function. Consumers solve

$$\max_{\{c_1, c_2\}} \theta_1 \log c_1 + \theta_2 \log c_2$$

Subject to budget constraint

$$p_1 c_1 + p_2 c_2 = wL$$

This is a **constrained optimization problem**, which we solve using the **Lagrangian method**

Constrained Consumer Problem

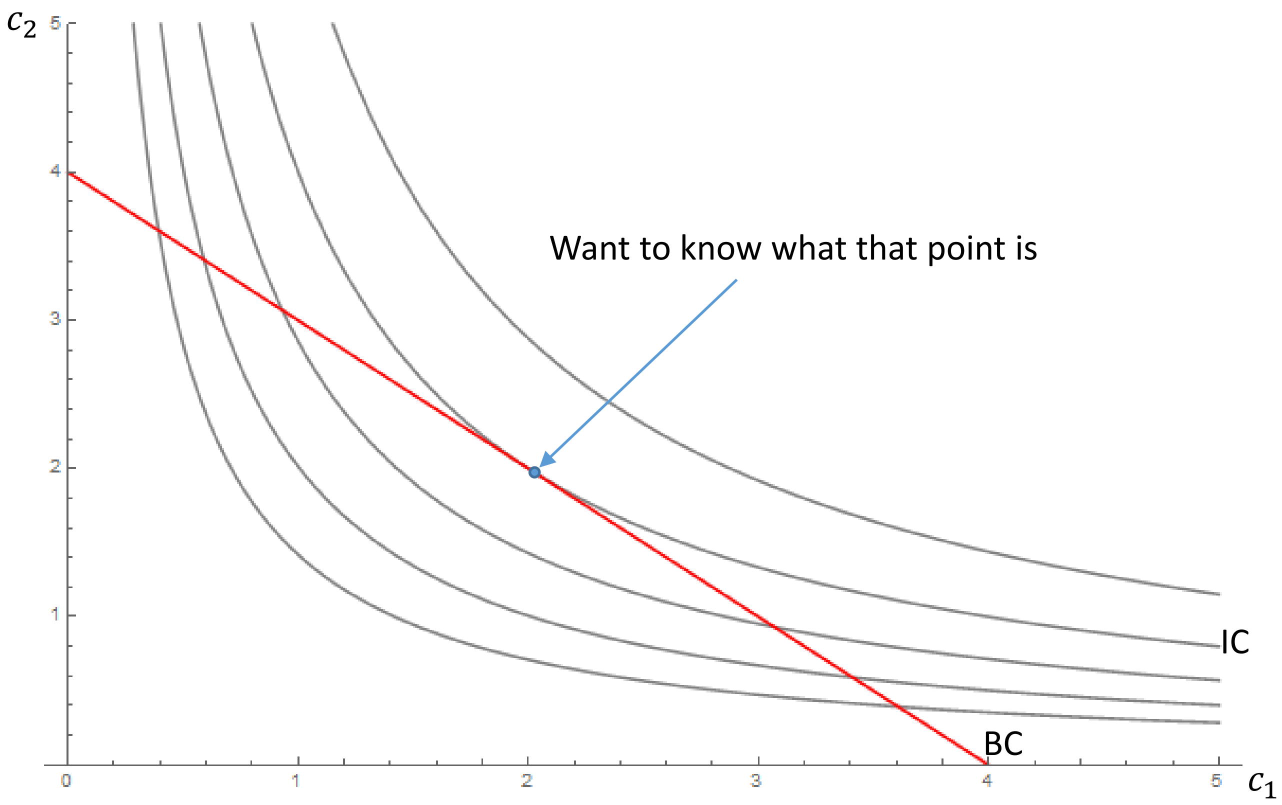
Problem is finding the Indifference Curve (IC) tangent to budget constraint

- From preferences: $U = \theta_1 \log c_1 + \theta_2 \log c_2$

- Therefore indifference curves (IC) given by (each IC corresponds to different value of U)

$$c_2 = \exp[(U - \theta_1 \log c_1)/\theta_2]$$

Constrained Consumer Problem: Graphically



Lagrangian Method for Constrained Optimization

Suppose we want to maximze the function $f(x,y)$ subject to $g(x,y) = 0$, the Lagrangian is

$$\mathcal{L}(x,y,\lambda) = f(x,y) + \lambda g(x,y)$$

Where λ is defined as the **lagrange multiplier**. We solve the maximization problem by finding $\{x, \mathrm{y}, \lambda\}$ that solve the following system of First Order Conditions (FOC):

$$\frac{\partial \mathcal{L}(x,y,\lambda)}{\partial x} = 0$$

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$$\frac{\partial \mathcal{L}(x, y, \lambda)}{\partial \lambda} = 0 \Rightarrow g(x, y) = 0$$

Lagrangian Method for Consumer Problem

The Lagrangian for the Consumer problem is

$$\mathcal{L}(c_1, c_2, \lambda) = \theta_1 \log c_1 + \theta_2 \log c_2 + \lambda(wL - p_1 c_1 - p_2 c_2)$$

The First Order Conditions (FOC) are:

$$[c_1]: \ \frac{\partial \mathcal{L}(c_1, c_2, \lambda)}{\partial c_1} = 0 \Rightarrow \frac{\partial(\theta_1 \log c_1 + \theta_2 \log c_2)}{\partial c_1} + \lambda \frac{\partial(wL - p_1 c_1 - p_2 c_2)}{\partial c_1} = 0$$

$$[c_2]: \ \frac{\partial \mathcal{L}(c_1, c_2, \lambda)}{\partial c_2} = 0 \Rightarrow \frac{\partial(\theta_1 \log c_1 + \theta_2 \log c_2)}{\partial c_2} + \lambda \frac{\partial(wL - p_1 c_1 - p_2 c_2)}{\partial c_2} = 0$$

$$[\lambda]: \ \frac{\partial \mathcal{L}(c_1, c_2, \lambda)}{\partial \lambda} = 0 \Rightarrow wL - p_1 c_1 - p_2 c_2 = 0$$

Lagrangian Method for Consumer Problem

The Lagrangian for the Consumer problem is

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The First Order Conditions (FOC) are:

$$[c_1]: \frac{\partial(\theta_1 \log c_1 + \theta_2 \log c_2)}{\partial c_1} + \lambda \frac{\partial(wL - p_1 c_1 - p_2 c_2)}{\partial c_1} = 0 \Rightarrow \frac{\theta_1}{c_1} - \lambda p_1 = 0$$

$$[c_2]: \frac{\partial(\theta_1 \log c_1 + \theta_2 \log c_2)}{\partial c_2} + \lambda \frac{\partial(wL - p_1 c_1 - p_2 c_2)}{\partial c_2} = 0 \Rightarrow \frac{\theta_2}{c_2} - \lambda p_2 = 0$$

$$[\lambda]: \frac{\partial \mathcal{L}(c_1, c_2, \lambda)}{\partial \lambda} = 0 \Rightarrow wL - p_1 c_1 - p_2 c_2 = 0$$

Solving the System of Equations

Have a system of equations with three equations and three unknowns $\{c_1, c_2, \lambda\}$

- Same number of unknowns and equations, so should have unique solution
- Note: For unique solution, can't have one equation be a combination of other equations

Solving the System of Equations

System is (equations from FOC, moved the negatives to other side):

$$\frac{\theta_1}{c_1} = \lambda p_1$$

$$\frac{\theta_2}{c_2} = \lambda p_2$$

$$wL = p_1 c_1 + p_2 c_2$$

Using the first two FOC together, we can find relative consumption (λ disappears)

$$\frac{\left(\frac{\theta_1}{c_1}\right)}{\left(\frac{\theta_2}{c_2}\right)} = \frac{\lambda p_1}{\lambda p_2} \Rightarrow \frac{\theta_1}{\theta_2}\frac{c_2}{c_1} = \frac{p_1}{p_2} \Rightarrow c_2 = \frac{\theta_2}{\theta_1}\frac{p_1}{p_2}c_1$$

Solving the System of Equations

Now take the relative productivities

$$c_2 = \frac{\theta_2}{\theta_1}\frac{p_1}{p_2}c_1$$

and combine it with the budget constraint to get

$$wL = p_1 c_1 + p_2\left(\frac{\theta_2}{\theta_1}\frac{p_1}{p_2}c_1\right) \Rightarrow wL = p_1 c_1 + \left(\frac{\theta_2}{\theta_1}p_1 c_1\right)$$

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Rearranging the above gives c_1

$$c_1 = \frac{wL}{p_1}\left(\frac{\theta_1}{\theta_1 + \theta_2}\right)$$

Which we can plug back in to relative productivites to get c_2

Solution to Consumers Problem

Therefore, given prices and wages, solution to consumers problem is

$$c_1 = \frac{wL}{p_1}\left(\frac{\theta_1}{\theta_1 + \theta_2}\right)$$

$$c_2 = \frac{wL}{p_2}\left(\frac{\theta_2}{\theta_1 + \theta_2}\right)$$

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$$c_2 = \frac{wL}{p_2}\left(\frac{\theta_2}{\theta_1 + \theta_2}\right)$$

Note: Cobb-Douglas preferences imply constant expenditure shares.

$$\frac{p_1 c_1}{wL} = \left(\frac{\theta_1}{\theta_1 + \theta_2}\right)$$

- Consumption of good 1 doesn't depend on price of good 2.

Solving For the Equilibrium: Firm Problem

Firms that produce good m solve:

$$\max_{\{y_m, l_m\}} p_m y_m - w_m l_m$$

Subject to their production function:

$$y_m = \frac{1}{a_m} l_m$$

Don't need to do Lagrangian

- Instead, substitute production function in for output in optimization problem
- Can take this shortcut since no meaningful tradeoffs. If we know $l_m \Rightarrow$ we know y_m.
- We can do Lagrangian if we want. Get same answer.

Solving For the Equilibrium: Firm Problem

Firms that produce good m solve:

$$\max_{\{l_m\}} p_m \left(\frac{1}{a_m} l_m \right) - w_m l_m$$

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Again, solve using FOCs

$$[l_m]\colon \frac{\partial \left(p_m \left(\frac{1}{a_m} l_m \right) - w_m l_m \right)}{\partial l_m} = 0$$

Solving For the Equilibrium: Firm Problem

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$$\max_{\{l_m\}} p_m \left(\frac{1}{a_m} l_m \right) - w_m l_m$$

Again, solve using FOCs

$$[l_m]: \frac{\partial \left(p_m \left(\frac{1}{a_m} l_m \right) - w_m l_m \right)}{\partial l_m} = 0 \Rightarrow \frac{\overbrace{p_m}^{MR}}{a_m} = \overbrace{w}^{\text{MC}}$$

- Marginal Revenue (MR) = Marginal Cost (MC)

Important!! This only works for interior solutions ($l_m > 0$). Need to worry about corner solutions separately ($l_m = 0$).

Solution to Firm's Problem

Interior Solution

- If $l_m > 0$, must have that $\frac{p_m}{a_m} = w$ in equilibrium

Corner Solution

- Firms don't choose prices. What if $\frac{p_m}{a_m} < w$?

Solution to Firm's Problem

Interior Solution

- If $l_m > 0$, must have that $\frac{p_m}{a_m} = w$ in equilibrium

Corner Solution

- Firms don't choose prices. What if $\frac{p_m}{a_m} < w$?
- Then firms won't produce good m in equilibrium.
- Let $\pi_m(l_m)$ be the function that represents firm m's profit when it uses l_m units of labor

$$\pi_m(l_m) = \overbrace{\left(\frac{p_m}{a_m} - w\right)}^{\text{Negative}} l_m; \text{ maximized at } \pi_m(0) = 0$$

Solution to Firm's Problem

Interior Solution

- If $l_m > 0$, must have that $\frac{p_m}{a_m} = w$ in equilibrium

Corner Solution

- What if $\frac{p_m}{a_m} < w$? Then firms won't produce good m in equilibrium, so $l_m = 0$
- What if $\frac{p_m}{a_m} > w$?

Solution to Firm's Problem

Interior Solution

- If $l_m > 0$, must have that $\frac{p_m}{a_m} = w$ in equilibrium

Corner Solution

- What if $\frac{p_m}{a_m} < w$? Then firms won't produce good m in equilibrium, so $l_m = 0$
- What if $\frac{p_m}{a_m} > w$? Will never happen in equilibrium.

$$\pi_m(l_m) = \overbrace{\left(\frac{p_m}{a_m} - w\right)}^{\text{Positive}} l_m; \text{ maximized at } \pi_m(\infty) = \infty$$

Firms will want to produce infinite amounts of output, prices will adjust so they can't.

Equilibrium Definition

Equilibrium is prices $\{p_1, p_2\}$, wages, $\{w_H, w_F\}$ and allocations$\left\{c_1^i, c_2^i; l_1^i, l_2^i; y_1^i, y_2^i\right\}_{i\in\{H,F\}}$ such

1. Consumers maximize utility

2. Firms maximize profits

3. Markets clear

Full System of Equations for Equilibrium

Equilibrium Unknowns: prices $\{p_1, p_2\}$, wages, $\{w_H, w_F\}$ and allocations $\{c_m^i; l_m^i; y_m^i\}_{\substack{i\in\{H,F\} \\ m\in\{1,2\}}}$

Equilibrium Equations

- Consumer Optimization in each country, $i = H, F$

$$c_1^i = \frac{w^i L^i}{p_1}\left(\frac{\theta_1}{\theta_1 + \theta_2}\right);\ c_2^i = \frac{w^i L^i}{p_2^i}\left(\frac{\theta_2}{\theta_1 + \theta_2}\right)$$

- Firm Optimization in each country, $i = H, F$, for each good, $m = 1,2$

$$\frac{p_m}{a_m^i} = w^i, \qquad \text{if } l_m^i > 0;\ \text{ production function: } y_m^i = \frac{1}{a_m} l_m^i$$

- Market clearing conditions for labor and goods

Solving Autarky Equilibrium

Usually easier to solve **autarky equilibrium**; start with that. Need to find prices $\{p_1, p_2\}$, wages, $\{w\}$ and allocations$\{c_1, c_2; l_1, l_2;\ y_1, y_2\}$ such that (suppress country superscripts)

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- Consumer Optimization holds

$$c_1 = \frac{wL}{p_1}\left(\frac{\theta_1}{\theta_1 + \theta_2}\right); c_2 = \frac{wL}{p_2}\left(\frac{\theta_2}{\theta_1 + \theta_2}\right)$$

- Firm Optimization for each good, $m = 1{,}2$ (+ production function $y_m = \frac{1}{a_m} l_m$)

$$\frac{p_m}{a_m} = w, \qquad \text{if } l_m > 0$$

- Market clearing conditions hold for labor: $l_1 + l_2 = L$, and for goods: $c_1 = y_1$, $c_2 = y_2$

Solving Autarky Equilibrium

Step 1: We know both goods will be consumed. Autarky, so both are produced. Plug goods clearing ($y_m = c_m$)and prices ($w = p_m/a_m$) from Firms into Consumer eqns:

$$y_1 = \frac{(p_1/a_1)L}{p_1}\left(\frac{\theta_1}{\theta_1 + \theta_2}\right); y_2 = \frac{(p_2/a_2)L}{p_2}\left(\frac{\theta_2}{\theta_1 + \theta_2}\right)$$

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Cancelling the prices gives our **allocations for goods**

$$y_1 = \frac{1}{a_1}L\left(\frac{\theta_1}{\theta_1 + \theta_2}\right); y_2 = \frac{1}{a_2}L\left(\frac{\theta_2}{\theta_1 + \theta_2}\right)$$

Solving Autarky Equilibrium

Step 2: Inserting allocations for goods into the production function gives **labor allocations**

$$l_m = a_m\left(\frac{1}{a_1}L\left(\frac{\theta_1}{\theta_1+\theta_2}\right)\right) \Rightarrow l_m = L\left(\frac{\theta_1}{\theta_1+\theta_2}\right), \qquad m = 1,2$$

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Step 3: When computing prices, **only relative prices matter**. We know **relative prices** (good prices relative to the wage) from rearranging the firm's FOC

$$\frac{p_m}{w} = a_m, \qquad m = 1,2$$

And we're done. Don't need to know w, since $w/w = 1$

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$$\frac{p_m}{w} = a_m, \qquad m = 1,2$$

And we're done. Don't need to know w, since $w/w = 1$. Typically normalize $w = 1$ to make things neater, in which case **prices** are

$$w = 1, \qquad p_1 = a_1, p_2 = a_2$$

What Did We Learn From Autarky Equilibrium?

- We learned how to solve the model in the simplest case

- We learned something about our assumptions. Cobb-Douglas preferences ⇒ constant expenditure shares and labor allocation doesn't depend on productivity of good

- We learned only relative prices matter. That implies we had 1 less equilibrium object than we thought (didn't need to solve for wages).

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- We learned something about our assumptions. Cobb-Douglas preferences ⇒ constant expenditure shares and labor allocation doesn't depend on productivity of good

- We learned only relative prices matter. That implies we had 1 less equilibrium object than we thought (didn't need to solve for wages).

- **Walras' Law** (loosely), in general equilibrium models, if all but one of the equilibrium constraints hold, then the last one will automatically.

- Notice we didn't need to use the labor market clearing condition

- We could have used the labor market clearing condition, but left out another. Basically, one of our prices doesn't matter, and one of our equilibrium equations holds for free.

Solving Free Trade Equilibrium

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How do we know second assumption will hold? We don't. But we can solve for implied equilibrium under complete specialization and verify it works

- We will get negative values somewhere if it doesn't

Additional Simplifying Assumptions

In Economics, very important to understand your model's assumptions.

What other simplifying assumptions did we make and why?

- Countries have same preferences. **Why:** Notational simplicity. Makes the algebra slightly simpler, since each country will have same relative consumption bundle.

Important to consider your question/goal before making assumptions. For this problem we are looking for a simple, stylized, GE model of trade we can build off of.