Two Countries: H & F

Cobb-Douglas Preferences: $\boldsymbol{\theta_0, \theta_1, \theta_2, \theta_3}$

Four Goods:

- **Good 0 nontraded, produced in each country**
- **Good 1 traded, produced by both countries (potentially)**
- **Good 2 traded, Home produced only, tariffs**
- **Good 3 traded, Foreign produced only, tariffs**

Unit Labor Requirements: $\boldsymbol{a_0^H, a_0^F, a_1^H, a_1^F, a_2^H, a_3^F}$

Fixed Labor supply

Note: You should know the difference between tariffs, iceburg costs, and quotas; but I won't have these in the define an equilibrium part of the exam. After defining the equilibrium with tariffs, I also define the equilibrium with free trade (which has the benefit of making prices be the same across countries).

Equilibrium is prices $\{p_0^i, p_1, p_2^i, p_3^i\}$, wages $\{w^i\}$ consumption allocations $\{c_0^i, c_1^i, c_2^i, c_3^i\}$, output allocations $\{y_0^i, y_1^i, y_2^{Hi}, y_3^{Fi}\}$, labor allocations $\{l_0^i, l_1^i, l_2^{Hi}, l_3^{Fi}\}$, and transfers $\{T^i\}$ for countries $i = H, F$ such that

1) Consumers Problem: Given prices $\{p_0^i, p_1, p_2^i, p_3^i\}$, wages $\{w^i\}$, and transfers T^i, consumers in country i maximize their utility

$$\max U(c_0^i, c_1^i, c_2^i, c_3^i) = \theta_0 \log c_0^i + \theta_1 \log c_1^i + \theta_2 \log c_2^i + \theta_3 \log c_3^i$$

Subject to their budget constraint

$$p_0^i c_0^i + p_1 c_1^i + p_2^i c_2^i + p_3^i c_3^i = w^i L^i + T^i$$

2) Firms Problem: Given prices $\{p_0^i, p_1, p_2^i, p_3^i\}$, wages $\{w^i\}$, and tariffs $\{\tau^H, \tau^F\}$, firms in country i maximize their profits

For good 0

$$\max p_0^i y_0^i - w^i l_0^i$$

Subject to their production function

$$y_0^i = \frac{1}{a_0^i} l_0^i$$

For good 1

$$\max p^1 y_0^i - w^i l_1^i$$

Subject to their production function

$$y_1^i = \frac{1}{a_1^i} l_1^i$$

For good 2 (Home Firms Only)

Domestic Profits: (there was a typo here, now corrected)

$$\max p_2^H y_2^{HH} - w^H l_2^{HH}$$

Subject to their production function

$$y_2^{HH} = \frac{1}{a_2^H} l_2^{HH}$$

Foreign Profits: (p_2^F is the price paid by consumers in the foreign country for good 2, note that we must have $p_2^F = \tau^F p_2^H$, where $(\tau^F - 1)$ is the tariff rate)

$$\max \frac{1}{\tau^F} p_2^F y_2^{HF} - w^H l_2^{HF}$$

Subject to their production function

$$y_2^{HF} = \frac{1}{a_2^H} l_2^{HF}$$

For good 3 (Foreign Firms Only)

You can write it exactly the same as for good 2 above, swapping H and F and replacing 2 with 3; or you can write it the condensed way below

Total Profits

$$\max p_3^F y_3^F - w^H l_3^F$$

Subject to their production function

$$y_3^F = \frac{1}{a_3^F} l_3^F$$

Where here $y_3^F = y_3^{FH} + y_3^{FF}$; and $l_3^F = l_3^{FH} + l_3^{FF}$. Here we are exploiting that $p_3^H = \tau^H p_3^F$.

3) Market Clearing

Market clearing for goods for $\boldsymbol{i = H, F}$

Good 0:

$$c_0^i = y_0^i$$

Good 1:

$$c_1^H + c_1^F = y_1^H + y_1^F$$

Good 2:

$$c_2^H = y_2^{HH}$$

$$c_2^F = y_2^{HF}$$

Good 3:

$$c_3^H = y_3^{FH}$$

$$c_3^F = y_3^{FF}$$

Labor Market Clearing for $\boldsymbol{i = H, F}$

Home:

$$l_0^H + l_1^H + l_2^{HH} + l_2^{HF} = L^H$$

Foreign:

$$l_0^F + l_1^F + l_3^{FH} + l_3^{FF} = L^F$$

Government Budget Balances for $\boldsymbol{i = H, F}$ ***(taxes = transfers)***

Home:

$$(\tau^H - 1)p_3^H y_3^{FH} = T^H$$

Foreign:

$$(\tau^F - 1)p_2^F y_H^{HF} = T^F$$

(Gov collects revenue on the good it imports from the other countries)

Endogeneous Variables: prices $\{p_0^i, p_1, p_2^i, p_3^i\}$, wages $\{w^i\}$ consumption allocations $\{c_0^i, c_1^i, c_2^i, c_3^i\}$, output allocations $\{y_0^i, y_1^i, y_2^{Hi}, y_3^{Fi}\}$, labor allocations $\{l_0^i, l_1^i, l_2^{Hi}, l_3^{Fi}\}$, and transfers $\{T^i\}$ for countries $i = H, F$

Exogeneous Varaibles: Total Labor Supplies $\{L^i\}$, labor productivities $\{a_0^i, a_1^i, a_2^H, a_3^F\}$, preference parameters $\{\theta_0, \theta_1, \theta_2, \theta_3\}$, tariff rates $\{\tau^i\}$

Variation with no tariffs. *Important changes: don't need anyway to separate out Home and Foreign markets for goods 2 and 3, since same price; essentially a single market.*

Equilibrium is prices $\{p_0^i, p_1, p_2, p_3\}$, wages $\{w^i\}$ consumption allocations $\{c_0^i, c_1^i, c_2^i, c_3^i\}$, output allocations $\{y_0^i, y_1^i, y_2^H, y_3^F\}$, and labor allocations $\{l_0^i, l_1^i, l_2^H, l_3^F\}$, for countries $i = H, F$ such that

1) Consumers Problem: Given prices $\{p_0^i, p_1, p_2^i, p_3^i\}$, wages $\{w^i\}$, and transfers T^i, consumers in country i maximize their utility

$$\max U\left(c_0^i, c_1^i, c_2^i, c_3^i\right) = \theta_0 \log c_0^i + \theta_1 \log c_1^i + \theta_2 \log c_2^i + \theta_3 \log c_3^i$$

Subject to their budget constraint

$$p_0^i c_0^i + p_1 c_1^i + p_2 c_2^i + p_3 c_3^i = w^i L^i + T^i$$

2) Firms Problem: Given prices $\{p_0^i, p_1, p_2, p_3\}$, wages $\{w^i\}$, firms in country i maximize their profits

For good 0

$$\max p_0^i y_0^i - w^i l_0^i$$

Subject to their production function

$$y_0^i = \frac{1}{a_0^i} l_0^i$$

For good 1

$$\max p^1 y_0^i - w^i l_1^i$$

Subject to their production function

$$y_1^i = \frac{1}{a_1^i} l_1^i$$

For good 2 (Home Firms Only)

Total Profits:

$$\max p_2 y_2^H - w^H l_2^H$$

Subject to their production function

$$y_2^H = \frac{1}{a_2^H} l_2^H$$

For good 3 (Foreign Firms Only)

You can write it exactly the same as for good 2 above, swapping H and F and replacing 2 with 3; or you can write it the condensed way below

Total Profits

$$\max p_3 y_3^F - w^H l_3^F$$

Subject to their production function

$$y_3^F = \frac{1}{a_3^F} l_3^F$$

3) Market Clearing

Market clearing for goods for $\boldsymbol{i = H, F}$

Good 0:

$$c_0^i = y_0^i$$

Good 1:

$$c_1^H + c_1^F = y_1^H + y_1^F$$

Good 2:

$$c_2^H + c_2^F = y_2^H$$

Good 3:

$$c_3^H + c_3^F = y_3^F$$

Labor Market Clearing for $\boldsymbol{i = H, F}$

Home:

$$l_0^H + l_1^H + l_2^H = L^H$$

Foreign:

$$l_0^F + l_1^F + l_3^F = L^F$$

Other Variations

Iceberg Trade Costs: Key difference versus tariffs is that instead of entering into the profit function, iceberg trade costs enter into the production function. There is also no government revenues/transfers. For example for good 2 produced in H for consumption in F:

Foreign Profits:

$$\max p_2^F y_2^{HF} - w^H l_2^{HF}$$

Subject to their production function

$$y_2^{HF} = \frac{1}{\tau^F a_2^H} l_2^{HF}$$

Quotas: Whereas iceberg trade costs and tariffs go in the firm's problem; quotas go in the market clearing section.

For example, if we had a quota on good 1 in H, it would mean

$$c_1^H - y_1^H \leq Quota$$

Since $c_1^H - y_1^H$ is H's imports of good 1.