ECO 442 Worksheet 1 Prof Jack Rossbach

Question 1: Ricardian Model with Transfers

Suppose we have the same two country, two good Ricardian model with as in class.

Consumer preferences are the same across countries and equal to

$$U^i(c_1^i, c_2^i) = \theta_1 \log c_1^i + \theta_2 \log c_2^i, \qquad i = H, F$$

Where $\theta_1, \theta_2 > 0$.

1.1) Suppose we have free trade and there is a flat transfer, T, from consumers in the Foreign country to consumers in the Home country. Write the budget constraint for consumers in each country.

Budget constraint for Home

$$p_1 c_1^H + p_2 c_2^H = w^H L^H + T$$

Budget constraint for Foreign

$$p_1 c_1^F + p_2 c_2^F = w^F L^F - T$$

1.2) What is the solution to the Consumer Problem in each country?

[Hint: the FOC wrt consumption are unchanged] Notationally, it can be helpful to let T^i represent the transfer to country i, which may be either negative or positive.

Let $T^H = T; T^F = -T$.

FOC for consumer's problem are

$$[c_1]:\ \theta_1 / c_1^i - \lambda p_1 = 0$$

$$[c_2]:\ \theta_2 / c_2^i - \lambda p_2 = 0$$

Along with Budget Constraint: $p_1 c_1^i + p_2 c_2^i = w^i L^i + T^i$

Step 1: Combine the FOC for consumption to get

$$\frac{\theta_1 / c_1^i}{\theta_2 / c_2^i} = \frac{\lambda p_1}{\lambda p_2} \Rightarrow c_2^i = \frac{\theta_2}{\theta_1}\frac{p_1}{p_2} c_1^i$$

Step 2: Plug into the Budget Constraint

$$w^i L^i + T^i = p_1 c_1^i + p_2 \left(\frac{\theta_2}{\theta_1}\frac{p_1}{p_2} c_1^i\right)$$

and simplify to get

$$c_1^i = \frac{w^i L^i + T^i}{p_1}\left(\frac{\theta_1}{\theta_1 + \theta_2}\right)$$

We can plug that into c_2^i to get a similar expression. In general we have, for $i = H, F; m = 1,2$

$$c_m^i = \frac{w^i L^i + T^i}{p_m}\left(\frac{\theta_m}{\theta_1 + \theta_2}\right)$$

1.3) Assume we have constant marginal input costs. Define an equilibrium for this economy. We no longer require balanced trade. What should the current account balance be in each country?

Equilibrium is prices $\{p_1, p_2\}$, wages, $\{w_H, w_F\}$ and allocations $\left\{c_m^i; l_m^i; y_m^i\right\}_{\substack{i\in\{H,F\}\\ m\in\{1,2\}}}$ such that:

1. Consumer Optimization in each country, $i = H, F$, for each good, $m = 1,2$:

$$c_m^i = \frac{w^i L^i + T^i}{p_m}\left(\frac{\theta_m}{\theta_1 + \theta_2}\right)$$

2. Firm Optimization in each country, $i = H, F$, for each good, $m = 1,2$

$$\frac{p_m}{a_m^i} = w^i, \text{ if } l_m^i > 0; \text{ production function: } y_m^i = \frac{1}{a_m} l_m^i$$

3. Labor market clearing for each country, $i = H, F$

$$l_1^i + l_2^i = L^i$$

and goods market clearing for each good, $m = 1,2$

$$c_m^H + c_m^F = y_m^H + y_m^F$$

4. Balanced Current Account. This says that Net Exports (Exports – Imports) equals Net Transfers (Transfers In – Transfers Out)

For Home country this is

$$\text{Exports} - \text{Imports} = \text{Transfers In}$$

Where Transfers In is Net Transfers [Since Transfers Out is zero]. Suppose Home country exports good 1 and imports good 2, then we can write it as

$$p_1 c_1^F + p_2 c_2^H = T$$

For Foreign country it's $\text{Exports} + \text{Imports} = -\text{Transfers Out}$ [since Transfers in is zero]

$$p_2 c_2^H + p_1 c_1^F = -T$$

1.4) Let $a_1^H = a_2^F = 1$ and $a_2^H = a_1^F = 2$ be the unit input costs. Let $L^H = L^F = 1$ be the labor supply for each country. Assume that in equilibrium we stay in the case with complete specialization, so Home produces only good 1 and Foreign produces only good 2. Let $\theta_1 = \theta_2 = 1$.

Normalize $w^H = 1$ and solve for this equilibrium. How does the transfer affect relative wages, relative prices, and relative consumption of each good? Is Home made better off by the transfer?

Step 1: Plug in equilibrium conditions to get labor allocations, output allocations, and price of good 1.

Step 2: Plug consumption from consumer problem into good 1 market clearing problem

$$\frac{w^H L^H + T^H}{p_1}\left(\frac{\theta_1}{\theta_1+\theta_2}\right) + \frac{w^F L^F + T^F}{p_1}\left(\frac{\theta_1}{\theta_1+\theta_2}\right) = y_1^H$$

Note that $T^H + T^F = 0$, so exactly same as slide 19 in lecture slides. Therefore relative wages are unchanged by transfers and given by (normalize $w_H = 1$):

$$w^F = \frac{L^H}{L^F}\left(\frac{\theta_2}{\theta_1}\right) = 1$$

Step 3: From FOC for firms in foreign we have that relative prices are unchanged also

$$\frac{p_2}{w_F} = a_2^F \Rightarrow p_2 = 1$$

Step 4: From consumer problem that means relative consumption is also unchanged. Indeed

$$c_1^i / c_2^i = 1$$

Step 5: Home country IS better off. World production unchanged by trade ($y_1^F = 1, y_2^F = 1$), however the Home country's share of world consumption in each good is equal to its share of world income

$$\frac{c_1^H}{y_1^H} = \frac{p_1 c_1^H}{p_1 y_1^H} = \frac{p_1 c_1^H}{p_1 c_1^H + p_1 c_1^F} =$$

$$= \frac{(w^H L^H + T^H)\big(\theta_1/(\theta_1+\theta_2)\big)}{(w^H L^H + T^H)\big(\theta_1/(\theta_1+\theta_2)\big) + (w^F L^F + T^F)\big(\theta_1/(\theta_1+\theta_2)\big)}$$

$$= \frac{w^H L^H + T^H}{w^H L^H + w^F L^F}$$

Plugging in wages, output, and labor shares gives

$$c_1^H = \frac{1+T}{2}$$

And in general, for country i (the foreign country obviously loses from the transfer)

$$c_m^i = \frac{1+T^i}{2}$$

1.5) Suppose the transfer is due to debt that the Foreign country owes the Home country. The Foreign country can choose to default, in which case it will not pay the transfer to the Home country. If Foreign country defaults, however, the Home country will retaliate by refusing to trade.

How high does the transfer have to be before the Foreign country would be better off defaulting? Consider using the following transformation for utility: $\text{Exp}\left[U^i\left(c_1^i, c_2^i\right)\right] = c_1^i c_2^i$.

Cobb-Douglas preferences, so constant expenditure shares. Autarky so consume what produce. Produce both goods, so from Firm FOC $p_1 = 2, p_2 = 1$ in Foreign country. Normalize $w_F = 1$.

$$c_m^F = \frac{w^F L^F}{p_m}\left(\frac{\theta_m}{\theta_1 + \theta_2}\right)$$

Therefore, plugging in prices and other set values gives

$$c_1^F = \frac{1}{4}, c_2^F = \frac{1}{2}$$

Which means transformed utility in autarky is

$$\text{Exp}[U^F(c_1^F, c_2^F)] = c_1^F c_2^F = \frac{1}{4}\frac{1}{2} = \frac{1}{8}$$

Under free trade, utility is

$$\text{Exp}[U^F(c_1^F, c_2^F)] = c_1^F c_2^F = \left(\frac{1-T}{2}\right)\left(\frac{1-T}{2}\right) = \frac{(1-T)^2}{4}$$

And so utility is higher in autarky if

$$\frac{1}{8} > \frac{(1-T)^2}{4}$$

$$\frac{1}{2} > (1-T)^2$$

We are only interested in the case where $T \in (0,1)$, so can focus on

$$\sqrt{\frac{1}{2}} > 1 - T$$

$$T > 1 - \sqrt{\frac{1}{2}} \approx 0.29$$

(other solution of quadratic equation is $T \approx 1.707$, which would be impossible for Foreign to pay).

1.7) Fill in the payoff matrix below, where Payoff $= \sqrt{4(\text{Exp}\left[U^i\left(c_1^i, c_2^i\right)\right]} = \sqrt{4c_1^i c_2^i}$

Note that if Home Refuses to Trade, we are in autarky and it is impossible for Foreign to Repay.

Payoff Matrix (F payoff, H payoff)	**Foreign Repays**	**Foreign Defaults**
Home Trades	$[(1-T), (\mathbf{1}+\boldsymbol{T})]$	$[\mathbf{1}, \mathbf{1}]$
Home Refuses to Trade	$\left[\sqrt{1/2}, \sqrt{1/2}\right]$	$\left[\sqrt{1/2}, \sqrt{1/2}\right]$

Is Home's threat of refusing to trade credible?

No. If Foreign Defaults, Home is still better off with Free Trade. The only NE is Foreign Defaults and Home Trades. Home always strictly prefers to trade. Foreign prefers to default or is indifferent.