ECON 442: Quantitative Trade Models

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Worksheet Discussion

Overview

- Introduce **Transfers** into standard 2x2 Ricardian model
- Observe that relative wages and prices weren't affected
- Solve a simple game involving default and punishment by refusing to trade

Will go over worksheet solution briefly (solution also posted on website)

- You should go over worksheets outside of class

Problem Setup

Consumer preferences are the same across countries and equal to

$$U^i\left(c_1^i, c_2^i\right) = \theta_1 \log c_1^i + \theta_2 \log c_2^i, \qquad i = H, F$$

Where $\theta_1, \theta_2 > 0$

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Important: $i = H, F$ is notation for saying the following two equations hold

$$U^H(c_1^H, c_2^H) = \theta_1 \log c_1^H + \theta_2 \log c_2^H\,, \qquad \text{(here } i = H\text{)}$$

$$U^F(c_1^F, c_2^F) = \theta_1 \log c_1^F + \theta_2 \log c_2^F\,, \qquad \text{(here } i = F\text{)}$$

- It's to save space and avoid writing nearly identical equations multiple times

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- What is budget constraint in each country?

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$$\text{Budget Constraint:} \quad \text{Expenditures} \leq \text{Income}$$

- Expenditures = spending on goods
- Income = labor income + transfers

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Where $\theta_1, \theta_2 > 0$

- Suppose there is a flat transfer $T > 0$ from Foreign to Home
- What is budget constraint in each country?

Budget Constraint in Home: $p_1 c_1^H + p_2 c_2^H = w^H L^H + \boldsymbol{T}$

Budget Constraint in Foreign: $p_1 c_1^F + p_2 c_2^F = w^F L^F - \boldsymbol{T}$

Solution to Consumer Problem

Consumer problem is maximize Utility, subject to budget constraint

- Write Lagrangian (let $T^H = T, T^F = -T$)

$$\mathcal{L}(c_1^i, c_2^i, \lambda) = \theta_1 \log c_1^i + \theta_2 \log c_2^i + \lambda\left(w^i L^i + T^i - p_1 c_1^i - p_2 c_2^i\right)$$

Only change in FOC is for budget constraint (since T^i does not change with c_1^i or c_2^i)

$$[c_1]: \frac{\theta_1}{c_1^i} - \lambda p_1^i = 0$$

$$[c_2]: \frac{\theta_2}{c_2^i} - \lambda p_2^i = 0$$

$$[\lambda]: \ w^i L^i + T^i - p_1 c_1^i - p_2 c_2^i = 0$$

Solution to Consumer Problem

Step 1: Combine the FOC for consumption to get

$$\frac{\theta_1/c_1^i}{\theta_2/c_2^i} = \frac{\lambda p_1}{\lambda p_2} \overbrace{\Rightarrow}^{\text{cancel } \lambda} \frac{\theta_1/c_1^i}{\theta_2/c_2^i} = \frac{p_1}{p_2} \overbrace{\Rightarrow}^{\frac{1}{\left(\frac{x}{y}\right)}=\frac{y}{x}} \frac{c_2^i}{\theta_2}\frac{\theta_1}{c_1^i} = \frac{p_1}{p_2}$$

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Then multiply both sides by $(\theta_2/\theta_1)c_1^i$

$$\left(\frac{\theta_2}{\theta_1}c_1^i\right)\frac{c_2^i}{\theta_2}\frac{\theta_1}{c_1^i} = \left(\frac{\theta_2}{\theta_1}c_1^i\right)\frac{p_1}{p_2} \overset{\text{cancel terms}}{\overbrace{\Rightarrow}} c_2^i = \left(\frac{\theta_2}{\theta_1}c_1^i\right)\frac{p_1}{p_2}$$

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Then plug into budget constraint

$$p_1 c_1^i + p_2\left(\left(\frac{\theta_2}{\theta_1}c_1^i\right)\frac{p_1}{p_2}\right) = w^i L^i + T^i \overset{\text{cancel and combine terms}}{\overbrace{\Rightarrow}} p_1 c_1^i\left(1 + \frac{\theta_2}{\theta_1}\right) = w^i L^i + T^i$$

Solution to Consumer Problem

Solving previous equation gives c_1^i

$$c_1^i = \frac{w^i L^i + T^i}{p_1}\left(\frac{\theta_1}{\theta_1 + \theta_2}\right)$$

Can plug back into equation for c_2^i to get

$$c_2^i = \frac{w^i L^i + T^i}{p_2}\left(\frac{\theta_2}{\theta_1 + \theta_2}\right)$$

Note: Only difference from basic model is the T^i in income

Definition of Equilibrium

Equilibrium is prices $\{p_1, p_2\}$, wages, $\{w\}$ and allocations$\{c_1, c_2; l_1, l_2;\ y_1, y_2\}$ such that

1. Consumers Optimize (maximize utility, subject to budget constraint)

2. Firms Optimize (maximize profits, subject to production function)

3. Market clearing for goods and labor

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4. (Optional) Balanced Current Account (Net Exports + Net Transfers = 0)

- Always require this (or something similar) holds in equilibrium
- Technically a combination of the other conditions (don't even need Walras' Law), so optional

Numerical Example

Let $\theta_1 = \theta_2 = 1$, $L^H = L^F = 1$, $a_1^H = a_2^F = 1$, and $a_2^H = a_1^F = 2$.

- Home has comparative advantage in good 1.

Assume complete specialization equilibrium holds (it will), what are values of goods?

- Plugging in values to eq'm conditions gives $y_1^H = y_2^F = 1$, and $p_1 = \mathrm{w}^{\mathrm{H}}$. Normalize $w^H = 1$

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- Note, from good 1 market clearing:

$$\frac{w^H L^H + T}{p_1}\left(\frac{\theta_1}{\theta_1 + \theta_2}\right) + \frac{w^F L^F - T}{p_1}\left(\frac{\theta_1}{\theta_1 + \theta_2}\right) = y_1^H$$

- The $+T$ and $-T$ cancel, so relative wages not affected by transfer

$$1\left(\frac{1}{2}\right) + \frac{w^F 1}{1}\left(\frac{1}{2}\right) = 1 \Rightarrow w^F = 1$$

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$$c_1^i / c_2^i = 1$$

- What does change?

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- What does change? **Home consumes bigger share of world output**

$$c_m^i = \frac{1 + T^i}{2}, \qquad \text{where} \begin{cases} T^i = T, & \text{if } i = H \\ T^i = -T, & \text{if } i = F \end{cases}$$

Debt Default and Exclusion from Markets

Having to pay transfer makes Foreign country worse off

- Suppose transfer is to repay debt. Foreign could **default** and not pay it.
- Home country **threatens no trade** if Foreign defaults
- How high does the transfer have to be for Foreign to want to default?

To answer this question need to know:

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To answer this question need to know:

1. Utility in autarky
2. Utility in trade, with the transfer

Utility in Autarky

We previously solved the model in autarky

$$c_1^F = \frac{1}{a_1^F} L^F \left(\frac{\theta_1}{\theta_1 + \theta_2} \right); c_2^F = \frac{1}{a_2^F} L^F \left(\frac{\theta_2}{\theta_1 + \theta_2} \right)$$

Plugging in $\theta_1 = \theta_2 = 1$, $L^F = 1$, $a_2^F = 1$, and $a_1^F = 2$ gives

$$c_1^F = \frac{1}{4}; c_2^F = \frac{1}{2}$$

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Plugging in $\theta_1 = \theta_2 = 1$, $L^F = 1$, $a_2^F = 1$, and $a_1^F = 2$ gives

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Useful to consider transformation of utility:

$$\text{Exp}[4 \times U^F(c_1^F, c_2^F)] = 4 c_1^F c_2^F = 4\left(\frac{1}{4}\right)\left(\frac{1}{2}\right) = \frac{1}{2}$$

Utility with Trade and Transfers

We also solved the model with Trade and Transfers for our specific parameter values

$$c_1^F = \frac{1-T}{2}; c_2^F = \frac{1-T}{2}$$

Transformed utility is:

$$\text{Exp}[4 \times U^F(c_1^F, c_2^F)] = 4c_1^F c_2^F = 4\left(\frac{1-T}{2}\right)\left(\frac{1-T}{2}\right) = (1-T)^2$$

When Should Foreign Default

Foreign should default when

$$\text{Utility in Autarky} > \text{Utility with Trade and Transfer}$$

$$\frac{1}{2} > (1-T)^2 \overset{\overbrace{\text{take } \sqrt{}}}{\Rightarrow} \sqrt{1/2} > (1-T) \overset{\overbrace{\substack{\text{flip } T \\ \text{and } \sqrt{1/2}}}}{\Rightarrow} T > 1 - \sqrt{1/2}$$

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- $1 - \sqrt{1/2} \approx 0.29$, so if have to transfer more than 29% of income, then default
- Unimportant: Other root to $\frac{1}{2} = (1-T)^2$ is $T = 1.707$, which we don't care about. Can't transfer more than 100% of income.

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Now we have a follow-up question

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- Consider following Payoff Matrix (transformed utility for each outcome)

Payoff Matrix (F payoff, H payoff)	Foreign Repays	Foreign Defaults
Home Trades	$[(1-T),(1+T)]$	$[1,1]$
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- If Foreign Repays, then Home should choose to Trade since $1 + T > \sqrt{1/2}$
- If Foreign Defaults, then Home should Trade, since $1 > \sqrt{1/2}$ (so not credible)

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- If Home Refuses to Trade, then Foreign indifferent (impossible to repay in autarky)
- If Home Trades, then Foreign should Default since $1 > (1-T)$

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- Using **Best Responses** found unique Nash Equilibrium
- Foreign Defaults and Home Trades anyways (punishment is an empty threat)

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- What does this say about commitment devices? [Dr Strangelove and the Bomb]