**ECON 256: Final Exam Practice Math/Formula Questions SOLUTIONS**

1. Given the following table, compute the mean value of $X$ (i.e. compute $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$)

| | Obs. 1 | Obs. 2 | Obs. 3 |
|---|---|---|---|
| ***X*** | −4 | 13 | 27 |

$$\bar{x} = \frac{1}{3}(-4 + 13 + 27) = \frac{1}{3}(36) = \mathbf{12}$$

2. Calculate the covariance between X and Y for the below table, where

$$\text{Cov}(X, Y) \equiv \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$$

Note that here $\bar{x} = 7$ and $\bar{y} = 4$.

| | Obs. 1 | Obs. 2 |
|---|---|---|
| ***X*** | 13 | 1 |
| ***Y*** | 2 | 6 |

$$\text{Cov}(X, Y) = \frac{1}{2-1}\big((13-7)(2-4) + (1-7)(6-4)\big)$$
$$= \frac{1}{1}\big((6)(-2) + (-6)(2)\big) = \big(-12 + (-12)\big) = \mathbf{-24}$$

3. The correlation coefficient between X and Y is defined as

$$r \equiv \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X)} \times \sqrt{\text{Var}(Y)}}$$

Suppose $Cov(X, Y) = 3$, while $Var(X) = 4$ and $Var(Y) = 9$. What is $r$ to 2 digits?

$$r = \frac{3}{\sqrt{4} \times \sqrt{9}} = \frac{3}{2 \times 3} = \frac{3}{6} = \mathbf{0.5}$$

4. Suppose we have the below regression relating hours spent procrastinating

$$\log(\text{Score}_i) = 95 - 3 \times \log(\text{Hours Procrastinating}_i) + \epsilon_i$$

If we increase hours spent procrastinating by 10%, by approximately what percent (percent, not percentage points) should we expect the score to rise/decline?

Important thing to note is that it's both log(score) and log(procrast.) in the regression. Therefore:

$$\text{% Change in Score} \approx -3 \times \text{% Change in Hours Procrastinating} = -3 \times 10 = \mathbf{-30}$$

So would expect a 30% decline in score

5. Suppose we have the following table

| Data Series | Observation 1 | Observation 2 | Observation 3 | Observation 4 | Observation 5 |
|---|---|---|---|---|---|
| X | 12 | 47 | 21 | 37 | 25 |
| Y | −8 | 15.5 | −39.5 | 41.5 | −7.5 |

which gives us the estimated regression coefficients plugged into the formula before

$$y_i = -30 + 0.5x_i + \epsilon_i$$

What is the predicted value of $Y$ when $X = 47$? What is the error for observation 2?

$$\hat{Y} = -30 + 0.5 \times 47 + 0 = -30 + 23.5 = \mathbf{-6.5}$$

What is the error for observation 2? Two ways, both give same answer:

Method 1: $\epsilon_i = y_i - (-30 + 0.5x_i) = y_i + 30 - 0.5x_i \Rightarrow$

$$\epsilon_2 = 15.5 + 30 - 0.5 * 47 = 45.5 - 23.5 = \mathbf{22}$$

Method 2: $\epsilon_i = y_i - \hat{Y}$, where $\hat{Y}$ is the predicted value of $Y$ when $X = x_i$. $\Rightarrow$

$$\epsilon_2 = 15.5 - (-6.5) = 15.5 + 6.5 = \mathbf{22}$$

6. For the above regression, how much would we expect $Y$ to change between $X = 12$ and $X = 47$? How much did it actually change by?

$$\text{Predicted Change} = \Delta\hat{Y} = \beta_X(X' - X) = 0.5 \times (47 - 12) = 0.5 \times 35 = \mathbf{17.5}$$

$$\text{Actual Change} = y_2 - y_1 = 15.5 - (-8) = 15.5 + 8 = \mathbf{23.5}$$

7. In the above regression we have an estimated slope coefficient $\beta_X = 0.5$. The standard error of the estimate is approximately 0.97. What is the 95% confidence interval for $\beta_X$? Is $\beta_X$ statistically significant?

95 % confidence interval for $\beta_X$ is approximately

$$[\beta_X - 2 \times SE\ , \beta_X + 2 \times SE\ ]$$
$$= [0.5 - (2 \times 0.97), 0.5 + (2 \times 0.97)]$$
$$= [0.5 - 1.94\ , 0.5 + 1.94]$$
$$= [\mathbf{-1.44}, \mathbf{2.44}]$$

The estimate is **NOT** statistically significant. This is because zero falls within the confidence interval (equivalently, the confidence interval contains both a positive and a negative number; indicating we are less than 95% sure that the number is positive)

8. Suppose we have the following table

| **Category** | Millions of People |
|---|---|
| Number of 12-19 Year Olds Enrolled in Secondary School | 5 |
| Number of **Non-**12-19 Year Olds Enrolled in Secondary School | 2 |
| Population of 12-19 Year Olds | 11 |
| Total Population | 30 |

What is the Gross Secondary School Enrollment Rate in percent?

$$\text{Gross Enrollment Rate} = \frac{\text{\#People Enrolled in Secondary School}}{\text{Population of } 12-19 \text{ Year Olds}} = \frac{5+2}{11} = 0.63 = \mathbf{63}\%$$

9. For the above table, what is the Net Secondary School Enrollment Rate in percent?

$$\text{Net Enrollment Rate} = \frac{\text{\#12-19 Year Olds in Secondary School}}{\text{Population of } 12-19 \text{ Year Olds}} = \frac{5}{11} = 0.45 = \mathbf{45}\%$$

10. Suppose a bank pays depositors 2%. How high of an interest rate would a bank have to charge on a loan to break even if there is only a 50% chance of full repayment? Suppose that if a borrower defaults, they have zero liability and pay back nothing. Recall the simple formula for breakeven interest rate with no liability is given by:

$$i = \frac{100 + r}{p} - 100$$

This means $r = 2$ (2% deposit rate) and $p = 0.5$ (50% repayment rate). Therefore break even interest rate is:

$$i = \frac{(100+2)}{0.5} - 100 = 204 - 100 = 104$$

So the bank must charge a **104% interest rate.** I.e. this means the interest rate must be 74 times larger than the deposit rate for the bank to break even.

11. Suppose we have the same set-up as the previous question, except now if a borrower defaults, they still pay back 30% of the initial loan. Recall the general formula for the breakeven interest rate is given by:

$$i = \frac{100 + r - (1-p)l}{p} - 100$$

Therefore break even interest rate is:

$$i = \frac{(100+2) - (1-0.5)30}{0.5} - 100 = \frac{102 - 15}{0.5} - 100 = 174 - 100 = 74$$

So the bank must charge a **74% interest rate;** which is still extremely high, but lower than when borrowers pay back nothing in the case of default.

12. Suppose a borrower has zero liability if a project fails. Suppose the borrower invests in a risky project where there is a 30% chance of failure where the borrower receives nothing. There is also a 70% chance of success where the borrower gets 5 times their investment.

Suppose the interest rate is 50%. What is the expected return for a borrower who borrows $100 to invest in a risky project? What is the expected return for the bank giving the loan?

Would both the bank and borrower want this loan to happen with the given interest rate?

For Borrower: If project succeeds (70% chance), project pays 500. 100 goes back to original loan, 50 goes to interest; leaving borrower with 350 profit.

$$\text{Return for Borrower} = 0.3 \times 0 + 0.7 \times 350 = \mathbf{245}$$

For Consumer: If project succeeds (70% chance), bank gains 50 (original 100 doesn't count as a gain). If project fails, bank loses initial loan and receives no interest, so loses 100.

$$\text{Return for Bank} = 0.3 \times -100 + 0.7 \times 50 = -30 + 35 = \mathbf{5}$$

**Yes, the loan would happen**, since both the Borrower and Bank have a positive expected return.