ECON 256 – Worksheet 1 **Professor Jack Rossbach**

Problem 1

Below is GDP per capita (PPP) for the United States and India in 1990 and 2005 from the World Development Indicators database.

GDP per capita (PPP, Constant 2011 International $)

	1990	**2005**
India	1,770	3,210
United States	37,060	49,760

Using these numbers answer the following questions:

1.1) Compute the average annual percent growth rate of GDP per capita (PPP) for the United States and India between 1990 and 2005. Round your answer to one decimal point.

Given a constant annual percentage growth rate, r, the formula for total growth rate is:

$$\text{Total Growth Rate after 15 years } = 100 * \left(\left[\frac{100 + r}{100}\right]^{15} - 1\right)$$

Inverting formula gives the average annual percent growth rate, r, in India and the U.S. respectively:

$$r = 100 * \left[\left(\frac{\text{value in 15 years}}{\text{value this year}}\right)^{\frac{1}{N}} - 1\right] = 100 * \left[\left(\frac{3210}{1770}\right)^{\frac{1}{15}} - 1\right] = 4.0 \text{ percent}$$

$$r = 100 * \left[\left(\frac{\text{value in 15 years}}{\text{value this year}}\right)^{\frac{1}{N}} - 1\right] = 100 * \left[\left(\frac{49760}{37060}\right)^{\frac{1}{15}} - 1\right] = 2.0 \text{ percent}$$

1.2) Suppose both countries continue growing at their current rates for another 25 years after 2005. What will GDP per capita be in each country in 2030? How much higher will GDP per capita be in the United States be in 2030 in proportion to GDP per capita in India.

Assume the current growth rate for India and the U.S. to be 4 percent and 2 percent respectively:

$$\text{Total Growth Rate after 25 years in India}$$

$$= 100 * \left(\left[\frac{100 + 4}{100}\right]^{25} - 1\right) = 166.6 \text{ percent}$$

$$\text{GDP per capital in India in } 2030 = 3210 * \left(\frac{100 + 166.6}{100}\right) = 8557$$

$$\text{Total Growth Rate after 25 years in the U.S.}$$

$$= 100 * \left(\left[\frac{100 + 2}{100}\right]^{25} - 1\right) = 64.1 \text{ percent}$$

$$\text{GDP per capital in the U.S. in } 2030 = 49760 * \left(\frac{100 + 64.1}{100}\right) = 81656$$

GDP per capita be in the United States in 2030 in proportion to GDP per capita in India

$$= \frac{81656}{8557} = 9.5$$

1.3) Assuming their current growth rates continue, how many years from 2030 will it take for the two countries to converge so that GDP per capita is the same in each country? Round to the nearest integer.

Use the formula of method 1 in notes and assume the US to be country 1 with growth rate 2 percent and India to be country 2 with growth rate 4 percent:

$$N = \frac{\log\left(\frac{\text{Country 2 GDP now}}{\text{Country 1 GDP now}}\right)}{\log\left(\frac{100 + r_1}{100 + r_2}\right)} = \frac{\log\left(\frac{8557}{81656}\right)}{\log\left(\frac{100 + 2}{100 + 4}\right)} = \frac{\log(0.104793)}{\log(0.980769)} = 116 \text{ years}$$

Note that it doesn't matter if Country 2 is the rich country or the poor country, we get the same answer either way. In the above equation we had India be Country 2, in the below equation we will have India be Country 1.

$$N = \frac{\log\left(\frac{\text{Country 2 GDP now}}{\text{Country 1 GDP now}}\right)}{\log\left(\frac{100 + r_1}{100 + r_2}\right)} = \frac{\log\left(\frac{81656}{8557}\right)}{\log\left(\frac{100 + 4}{100 + 2}\right)} = \frac{\log(9.54297)}{\log(1.019608)} = 116 \text{ years}$$

It also doesn't matter if you use log base 10 or the natural log in this particular formula. It can be important to carry your decimal places in intermediate steps, however.

$$\frac{\log(0.104793)}{\log(0.980769)} = 116.1683; \text{ but } \frac{\log(0.10)}{\log(0.98)} = 113.97$$

So make sure you don't round your numbers in the middle of calculations.