ECON 442: Quantitative Trade Models

Jack Rossbach

Why do Countries Trade?

Ricardian: Countries have different technologies for producing goods. Comparative advantage.

Heckscher-Ohlin: Countries have same technology for producing goods, but different factor endowments which are used in the production of the goods.

Armington: Countries have different goods, and consumers like to consume foreign goods.

Monopolistic Competition: Countries have firms which produce differentiated varieties of a good. Consumers like to consumer different varieties.

Increasing Returns: Cheaper to produce a good all in a single place, so countries should specialize and trade.

Armington (1969) Framework

- Countries produce different goods and want to consume all types of goods
- Comparative advantage imposed in an ad-hoc way
    - In previous models all countries could produce all goods, just at different costs
    - In this model, countries cannot produce other countries‘ goods
    - Japanese cars are different than American cars and the U.S. cannot produce Japanese cars (Toyota factories in U.S. are different than than Toyota factories in Japan)
- Model is relatively simple and easy to calibrate making it commonly used
    - Simple because because trade doesn‘t impact specialization in same way as previous frameworks

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  - Simple because because trade doesn‘t impact specialization in same way as previous frameworks
  - Later we‘ll see that many other models are actually equivalent to an Armington framework

Basic Setup

- $i, j = 1,2, \dots, J$ Countries (switching from H and F to numbers, since muliple countries)
- $m = 1, \dots, M$ sectors
- Each country produces it's own country specific good in each sector
- Consumers in country i have Constant Elasticity of Substitution (CES) preferences over goods

$$U^i = \sum_{m=1}^{M} \theta_m^i \log\left(\sum_{i=1}^{N} \mu_{mj}^i \left(c_{mj}^i\right)^{\rho}\right)$$

θ_m^i is sector m's expenditure share in country i. μ_{mj}^i is a taste shifter, which indicates how much consumers in country i want to consume sector m output from country j. $\sigma = \frac{1}{1-\rho}$ is the elasticity of substitution (typically $0 < \rho < 1$ so $\rho \uparrow \Rightarrow \sigma \uparrow$).

Equilibrium

Equilibrium is prices $\{p_{mj}^i\}$, wages $\{w^i\}$, allocations $\{c_{mj}^i, l_{mj}^i, y_{mj}^i\}$ such that

1) Given prices and wages, Consumer‘s maximize utility subject to their budget constraint

2) Given prices and wages, firm‘s maximize profits subject to their production functions

3) Markets Clear

1. Consumer's Problem

Consumers in country i solve

$$\max_{\{c_{mj}^i\}} \sum_{m=1}^{M} \theta_m^i \log\left(\sum_{i=1}^{N} \mu_{mj}^i \left(c_{mj}^i\right)^{\rho}\right)$$

subject to budget constraint:

$$\sum_{i=1}^{N} p_{mj}^i c_{mj}^i = w^i L^i$$

1. Household Demand: CES Price Index

If we set up and solve the Lagrangian, we'll have the solution to the consumer problem is

$$c_{mj}^{i} = \left(\mu_{mj}^{i}\right)^{\frac{1}{1-\rho}} \left(\frac{p_{mj}^{i}}{P_{m}^{i}}\right)^{\frac{1}{1-\rho}} C_{m}^{i}$$

Where P_m^i is a sector Price index

$$P_{m}^{i} = \left(\sum_{i=1}^{N} \left(\mu_{mj}^{i}\right)^{\frac{1}{1-\rho}} \left(p_{mj}^{i}\right)^{\frac{-\rho}{1-\rho}}\right)^{-\left(\frac{1-\rho}{\rho}\right)}$$

And C_m^i a sector Consumption index based on the Cobb-Douglas Expenditure Shares

$$C_{m}^{i} = \left(\frac{w^{i} L^{i}}{P_{m}^{i}}\right)\left(\frac{\theta_{m}^{i}}{\sum_{m=1}^{M} \theta_{m}^{i}}\right)$$

2. Firms Problem

Since we have constant returns to scale, we can separate firm's problem by destination.

Given $p_{m,i}^{j}, w^{i}$ firms in country i maximize profits for good m for destination j

$$\max p_{mi}^{j} y_{mi}^{j} - w^{i} l_{mi}^{j}$$

subject to production technology:

$$y_{mi}^{j} = z_{mi} l_{mi}^{j}$$

Firm Optimization Yields: $p_{m,i}^{j} = \frac{w^{i}}{z_{m,i}}$ if $y_{m,i}^{j} > 0$

3. Market Clearing

Goods market clears for each good. We'll just separate it by origin and destination.

$$c_{mi}^{j} = y_{mi}^{j}, \qquad m = 1, \dots, M; i, j = H, F$$

Labor market clears for each country

$$\sum_{j=H,F} \sum_{m=1}^{M} l_{mi}^{j} = L^{i}, \qquad i = H, F$$

Exogenous Variables

The exogenous variables in this model are

- Labor supplies: L^i
- Cobb-Douglas Sector Expenditure Shares: θ_m^i
- Countries-Specific Taste Shifters: μ_{mj}^i
- Elasticity of substititon/love for variety: ρ
- Productivity for production functions: z_{mi}

Next step will be to talk about calibrating these parameters using data