

ECO 442: Quantitative Trade Models

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Recap and Roadmap

What have we done so far?

- Simple graphical analysis of trade with comparative advantage
- Formalized the model and developed a precise definition of an equilibrium in the model
- Solved the model in autarky

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Where are we going?

- Solve the model with trade
- Circle back to the graphical analysis and compare it with our formalized model
- Start building on the model, which will allow us to do policy analysis

Longer Term

What are the ultimate goals in this class?

- Understand how to develop quantitative predictions using models
- Learn to estimate the exogeneous model parameters using data
- Start solving models computationally

Walras' Law

Walras' Law (loosely), in general equilibrium models, if all but one of the equilibrium equations hold, then the last equilibrium equation will hold automatically

- Observed Walras' Law when solving the autarky equilibrium, as we didn't use labor market clearing
- Essentially means we have one less equilibrium equation than we thought (It does still have to hold in equilibrium, it just does so automatically)
- We also have one less equilibrium unknown (endogeneous variable) than we thought, since only relative prices matter for allocations. Typically normalize wage equal to 1, but it doesn't matter which price we normalize or what value we set it equal to
- Note: A common mistake when putting models on the computer is to forget Walras' law

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Solving Free Trade Equilibrium

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 2. Suppose we will have complete specialization in equilibrium

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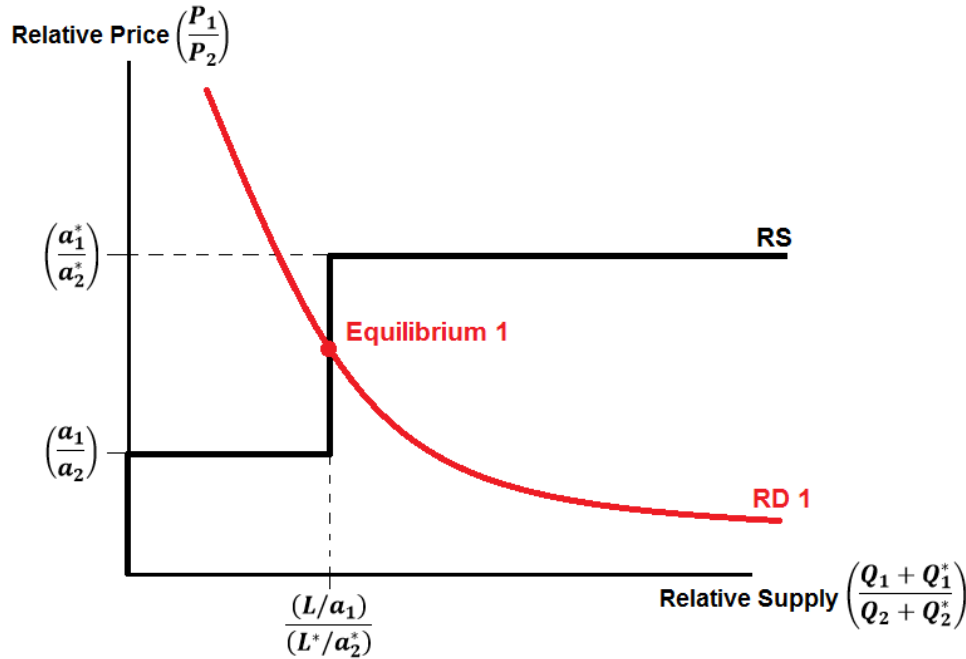
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How do we know second assumption will hold? We don't. But we can solve for implied equilibrium under complete specialization and verify it works

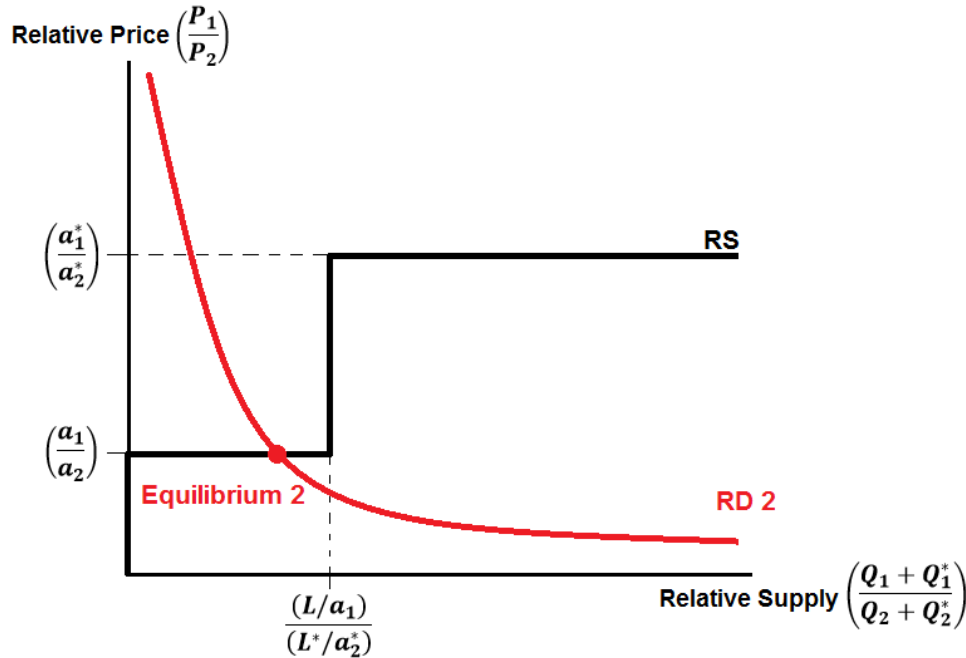
- We will get values inconsistent with an equilibrium if it doesn't

Finding Equilibrium using Relative Demand



- We solve model assuming equilibrium will be consistent with the above pattern of complete specialization.
- Afterwards we can check whether this assumption was valid (think of it as guess and check).

Finding Equilibrium using Relative Demand



- Possible that true equilibrium is supposed to be like this (Home produces both goods, foreign produces only good 2), in which case we will see our solution isn't a valid equilibrium.
- Rigorous way is to solve model for all five patterns of specialization (we don't, to save time)

Additional Simplifying Assumptions

In Economics, very important to understand your model's assumptions.

What other simplifying assumptions did we make and why?

- Countries have same preferences. **Why:** Notational simplicity. Makes the algebra slightly simpler, since each country will have same relative consumption bundles.
- Important to consider your question/goal before making assumptions. For this problem we are looking for a simple, stylized, GE model of trade we can build off of.

Equilibrium Definition

Equilibrium is prices $\{p_1, p_2\}$, wages, $\{w^H, w^F\}$ and allocations $\{c_1^i, c_2^i; l_1^i, l_2^i; y_1^i, y_2^i\}_{i \in \{H, F\}}$ s.t.

1. Consumers maximize utility
2. Firms maximize profits
3. Markets clear

Full System of Equations for Equilibrium

Equilibrium Unknowns: prices $\{p_1, p_2\}$, wages, $\{w^H, w^F\}$ and allocations $\{c_m^i; l_m^i; y_m^i\}_{\substack{i \in \{H, F\} \\ m \in \{1, 2\}}}$

Equilibrium Equations

- Consumer Optimization in each country, $i = H, F$

$$c_1^i = \frac{w^i L^i}{p_1} \left(\frac{\theta_1}{\theta_1 + \theta_2} \right); \quad c_2^i = \frac{w^i L^i}{p_2} \left(\frac{\theta_2}{\theta_1 + \theta_2} \right)$$

- Firm Optimization in each country, $i = H, F$, for each good, $m = 1, 2$

$$\frac{p_m}{a_m^i} = w^i, \quad \text{if } l_m^i > 0; \quad \text{production function: } y_m^i = \frac{1}{a_m^i} l_m^i$$

Market clearing conditions for labor and goods on next slide

Full System of Equations for Equilibrium

Equilibrium Equations

- Labor market clearing for each country, $i = H, F$

$$l_1^i + l_2^i = L^i$$

and goods market clearing for each good

$$\begin{array}{ccc} \text{World Consumption} & & \text{World Production} \\ \text{of Good 1} & & \text{of Good 1} \\ \hline c_1^H + c_1^F & = & y_1^H + y_1^F \end{array}$$

$$c_2^H + c_2^F = y_2^H + y_2^F$$

Solving Free Trade Equilibrium

Step 1: We are assuming complete specialization, so plug that into our equations

Home produces only good 1. Foreign produces only good 2.

- Market Clearing for Goods becomes

$$c_1^H + c_1^F = y_1^H$$

$$c_2^H + c_2^F = y_2^F$$

- Market Clearing for Labor becomes **equilibrium labor allocations**

$$l_1^H = L^H; \quad l_2^F = L^F$$

- Prices must satisfy

$$p_1 = a_1^H w^H; \quad p_2 = a_2^F w^F$$

Solving Free Trade Equilibrium

Step 2: Plug consumption from consumer problem into market clearing

- For Good 1, we have

$$\frac{w^H L^H \overbrace{\left(\frac{\theta_1}{\theta_1 + \theta_2} \right)}^{c_1^H}}{p_1} + \frac{w^F L^F \overbrace{\left(\frac{\theta_1}{\theta_1 + \theta_2} \right)}^{c_1^F}}{p_1} = y_1^H$$

Solving Free Trade Equilibrium

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- Plugging in prices

$$\overbrace{\frac{w^H L^H}{a_1^H w^H} \left(\frac{c_1^H}{\theta_1 + \theta_2} \right)} + \overbrace{\frac{w^F L^F}{a_1^H w^H} \left(\frac{c_1^F}{\theta_1 + \theta_2} \right)} = y_1^H$$

Solving Free Trade Equilibrium

$$\overbrace{\frac{w^H L^H}{a_1^H w^H} \left(\frac{\theta_1}{\theta_1 + \theta_2} \right)}^{c_1^H} + \overbrace{\frac{w^F L^F}{a_1^H w^H} \left(\frac{\theta_1}{\theta_1 + \theta_2} \right)}^{c_1^F} = y_1^H$$

Collecting terms, simplifying, and rearranging

$$\left(L^H + \frac{w^F}{w^H} L^F \right) \left(\frac{\theta_1}{\theta_1 + \theta_2} \right) = a_1^H y_1^H$$

Step 3: Plug production function + labor clearing into the above

$$\left(L^H + \frac{w^F}{w^H} L^F \right) \left(\frac{\theta_1}{\theta_1 + \theta_2} \right) = a_1^H \overbrace{\left(\frac{1}{a_1^H} L^H \right)}^{y_1^H}$$

Solving Free Trade Equilibrium

$$\left(L^H + \frac{w^F}{w^H} L^F \right) \left(\frac{\theta_1}{\theta_1 + \theta_2} \right) = a_1^H \overbrace{\left(\frac{1}{a_1^H} L^H \right)}^{y_1^H}$$

Canceling the a_1^H gives an expression that only depends on **relative wages**

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Solving Free Trade Equilibrium

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Dividing both sides by $(\theta_1/(\theta_1 + \theta_2))$

$$\left(L^H + \frac{w^F}{w^H} L^F\right) = L^H \left(\frac{\theta_1 + \theta_2}{\theta_1}\right)$$

Solving Free Trade Equilibrium

$$\left(L^H + \frac{w^F}{w^H} L^F \right) = L^H \left(\frac{\theta_1 + \theta_2}{\theta_1} \right)$$

Subtracting L^H from both sides

$$\frac{w^F}{w^H} L^F = L^H \left(\frac{\theta_1 + \theta_2}{\theta_1} \right) - L^H$$

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Simplifying

$$\frac{w^F}{w^H} L^F = L^H \left(\left(1 + \frac{\theta_2}{\theta_1} \right) - 1 \right)$$

$$\frac{w^F}{w^H} L^F = L^H \left(\frac{\theta_2}{\theta_1} \right)$$

Solving Free Trade Equilibrium

$$\frac{w^F}{w^H} L^F = L^H \left(\frac{\theta_2}{\theta_1} \right)$$

Dividing by both sides gives **relative wages** (here Foreign wages relative to Home wages)

$$\frac{w^F}{w^H} = \frac{L^H}{L^F} \left(\frac{\theta_2}{\theta_1} \right)$$

Solving Free Trade Equilibrium

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Relative Wages = 1/(Relative Labor Supply) × Relative Consumption Share of Exported Good

- A country's **relative wages decrease** as its **relative labor supply increases**
- A country's **relative wages increase** as **demand for its exported good increases**

Solving Free Trade Equilibrium

Lets use relative wages to get rest of equilibrium unknowns.

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Equilibrium relative prices are

- For good 1 (the good Home produces)

$$\frac{p_1}{w^H} = a_1^H$$

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- For good 2, we need to plug in relative wages in form of $w^F = w^H \frac{L^H}{L^F} \left(\frac{\theta_2}{\theta_1} \right)$, therefore

$$\frac{p_2}{\left(w^H \frac{L^H}{L^F} \left(\frac{\theta_2}{\theta_1} \right) \right)} = a_2^F \Rightarrow \frac{p_2}{w^H} = a_2^F \frac{L^H}{L^F} \left(\frac{\theta_2}{\theta_1} \right)$$

Solving Free Trade Equilibrium

We can then use prices to find **consumption allocations**

- Plugging in relative price for good 1 in Home country gives

$$c_1^H = \overbrace{(1/\alpha_1^H)}^{w^H/p_1} L^H \left(\frac{\theta_1}{\theta_1 + \theta_2} \right)$$

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- Likewise for good 2,

$$c_2^H = \underbrace{\left((1/a_2^F) \frac{L^F}{L^H} \left(\frac{\theta_1}{\theta_2} \right) \right)}_{\frac{w^H}{p_2} = \frac{w^F}{p_2} \times \frac{w^H}{w^F}} \underbrace{L^H \left(\frac{\theta_2}{\theta_1 + \theta_2} \right)}_{\text{Share of H income devoted to good 2}} = \underbrace{\frac{w^F/p_2}{(1/a_2^F)}}_{\text{Share of F income devoted to good 1}} L^F \left(\frac{\theta_1}{\theta_1 + \theta_2} \right)$$

The income share is just the theta term. Multiplying by $w^i L^i$ gives income allocated to good

Solving Free Trade Equilibrium

We can then use prices to find **consumption allocations**

- For Foreign country we follow the same process, and get

$$c_1^F = (1/a_1^H)L^H \left(\frac{\theta_2}{\theta_1 + \theta_2} \right), \quad c_2^F = (1/a_2^F)L^F \left(\frac{\theta_2}{\theta_1 + \theta_2} \right)$$

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That gives us all allocations, as well as all relative prices, so we're done

Balanced Trade

- Note that in a static model, we must have that **Trade is Balanced** for each country

$$\text{Value Exports} = \text{Value Imports}$$

Under of assumption of complete specialization (Home makes good 1, Foreign good 2)

$$\begin{array}{ccc} \text{Value of Home} & & \text{Value of Foreign} \\ \text{Imports} & & \text{Imports} \\ \underbrace{p_2 c_2^H} & = & \underbrace{p_1 c_1^F} \end{array}$$

- Note that **Home Imports = Foreign Exports** since only two countries
- In general, requires additional notation I don't want to introduce yet. For this model we don't need the balanced trade condition to solve the model, it's just something that holds.

Verifying Balanced Trade

We didn't use **balanced trade** to derive our analytical solution

- We can verify it holds in our equilibrium
- In general, verifying unused or intermediate equations such as with Walras' law or balanced trade is a great way to make sure your computer programs work

Verifying Balanced Trade

Recall, Balanced Trade equation under complete specialization is

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Lets **normalize** $w^H = 1$. Plugging in our prices and consumption allocations gives

$$\overbrace{a_2^F \frac{L^H}{L^F} \left(\frac{\theta_2}{\theta_1} \right)}^{p_2} \overbrace{(1/a_2^F) L^F \left(\frac{\theta_1}{\theta_1 + \theta_2} \right)}^{c_2^H} = \overbrace{a_1^H (1/a_1^H) L^H \left(\frac{\theta_2}{\theta_1 + \theta_2} \right)}^{p_1 c_1^F}$$

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Simplifying the above shows that it automatically holds at our equilibrium values

$$L^H \left(\frac{\theta_2}{\theta_1 + \theta_2} \right) = L^H \left(\frac{\theta_2}{\theta_1 + \theta_2} \right) \Rightarrow 1 = 1$$

Verifying Our Assumption of Complete Specialization

With our preferences and free trade, specialization and trade always an equilibrium

- Worst case scenario: countries are identical \Rightarrow no gains/losses from trade. Multiple equilibria in this case; specialization is one of them.

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- Worst case scenario: countries are identical \Rightarrow no gains/losses from trade. Multiple equilibria in this case; specialization is one of them.
- Not true when we add tariffs and trade costs. Then countries may choose not to trade.

Numerical Example

- Let's verify specialization holds in our free trade equilibrium
- In general, depends on the **exogenous parameters** of the model
- Set $L^H = L^F = 1$; $\theta_1 = \theta_2 = 1$. $a_1^H = a_2^F = 1$; $a_2^H = a_1^F = 2$.

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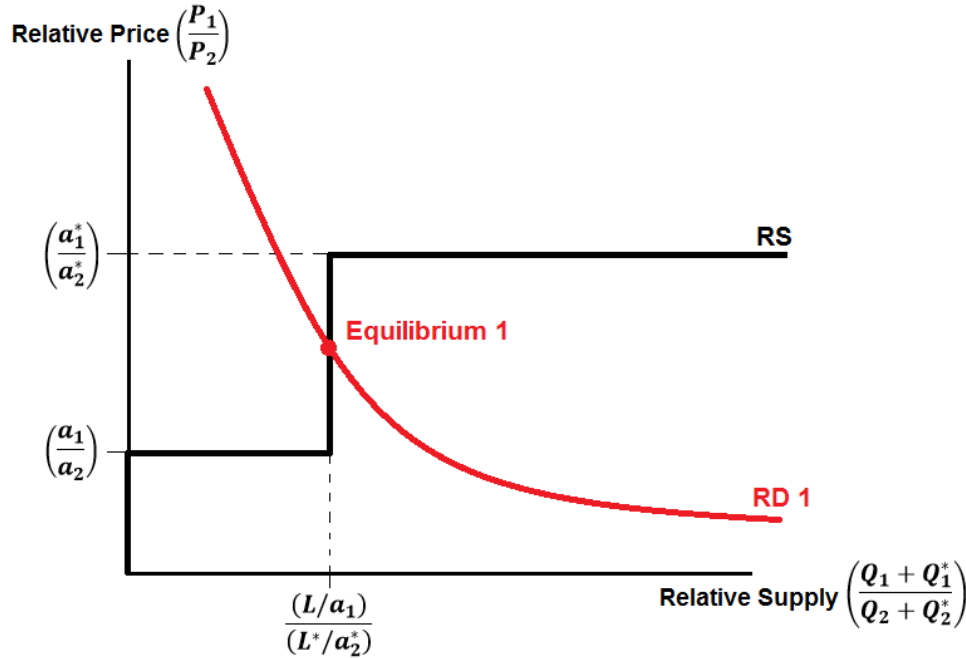
Relative unit input costs are

$$\frac{a_1^H}{a_2^H} = \frac{1}{2}; \frac{a_1^F}{a_2^F} = 2$$

Relative prices (now relative to each other) are

$$p_1 = a_1^H = 1 \text{ and } p_2 = a_2^F \frac{L^H}{L^F} \left(\frac{\theta_2}{\theta_1} \right) = 1 \Rightarrow \frac{p_1}{p_2} = 1$$

Finding Equilibrium using Relative Demand



$$\frac{1}{2} < 1 < 2 \Rightarrow \left(\frac{a_1^H}{a_2^H}\right) < \left(\frac{p_1}{p_2}\right) < \left(\frac{a_1^F}{a_2^F}\right) \Rightarrow \text{Home produces only good 1. Foreign produces only good 2.}$$

(Graph: no star = Home, star = Foreign)