ECO 745: Theory of International Economics

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Fall 2015 - Lecture 14

Gravity Models of Trade

Several of the models we have worked with have generated gravity equations

Perfect Competition:

- Armington (1969) framework, as shown by Anderson and van Wincoop (2003)
- Eaton-Kortum (2002) framework

Monopolistic Competition:

- Krugman (1980)
- Melitz (2003) and Chaney (2008)

Welfare in Gravity Models

Arkolakis, Constinot, Rodriguez-Clare (2012)

- How does the choice of model affect the implied gains from trade?
    - Answer: Not much
- Show that in all the previously mentioned frameworks, welfare gains from trade can be characterized by the share of domestic expenditure and the trade elasticity.
    - New micro-level models of trade, but same welfare implications as Armington models.
    - The good: Armington models are robust in their welfare predictions
    - The bad: Micro-level data is irrelevant for welfare gains from trade in standard frameworks

Recap of Armington Model

- $i = 1, \ldots, N$ countries, each produce a differentiated good according to $y_i = l_i$
- Labor endowment for country i is L_i
- Iceberg trade costs $\tau_{ij} \geq 1$, $\tau_{ii} = 1$. Cost to ship 1 unit of i's output to j: $w_i \tau_{ij}$
- Preferences are, $\sigma > 1$ is the elasticity of substitution,

$$U_i = \left(\sum_{i=1}^{N} (q_{ij})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

- Perfect competition, therefore price index in country j is

$$P_j = \left(\sum_{i=1}^{N} (w_i \tau_{ij})^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

Gravity in Armington Model

- Gravity equation generated by Armington model is $\left(Y_j = \sum_{i-1}^{N} X_{ij}\right)$:

$$X_{ij} = \left(\frac{w_i \tau_{ij}}{P_j}\right)^{1-\sigma} Y_j$$

- Therefore the trade share is given by

$$\lambda_{ij} := \frac{X_{ij}}{Y_j} = \left(\frac{w_i \tau_{ij}}{P_j}\right)^{1-\sigma} = \frac{\left(w_i \tau_{ij}\right)^{1-\sigma}}{\sum_{i=1}^{N} \left(w_i \tau_{ij}\right)^{1-\sigma}}$$

- Balanced trade implies $Y_j = w_j L_j$

- Trade elasticity is given by

$$\epsilon := \frac{\partial \log\left(X_{ij}/X_{jj}\right)}{\partial \tau_{ij}} = 1 - \sigma$$

Changes in Welfare

Suppose a shock changes countries ($\forall i \neq j$) labor endowments and trade costs with country j

What is the resulting change in real income $\left(W_j \coloneqq Y_j / P_j\right)$ for country j?

- Normalize $w_j = 1$. Note that $d \log Y_j = d \log w_j + d \log L_j = 0$, and therefore

$$d \log W_j = d \log Y_j - d \log P_j = -d \log P_j$$

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- From our derived price index we have:

$$\log P_j = \left(\frac{1}{1-\sigma}\right) \log\left(\sum_{i=1}^{N} \left(w_i \tau_{ij}\right)^{1-\sigma}\right) \Rightarrow$$

$$d \log P_j = \left(\frac{1}{1-\sigma}\right) \frac{\left[\sum_{i=1}^{N} \left(w_i \tau_{ij}\right)^{1-\sigma} (1-\sigma)\left(d \log \tau_{ij} + d \log w_i\right)\right]}{\sum_{i=1}^{N} \left(w_i \tau_{ij}\right)^{1-\sigma}} = \sum_{i=1}^{N} \lambda_{ij}\left(d \log \tau_{ij} + d \log w_i\right)$$

Changes in Welfare

Welfare changes given by

$$d \log W_j = -\sum_{i=1}^{N} \lambda_{ij} \left(d \log w_i + d \log \tau_{ij} \right)$$

Trade shares relative to domestic expenditures are given by $\lambda_{ij}/\lambda_{jj} = \left(w_i \tau_{ij} \right)^{1-\sigma}$, therefore

$$d \log \lambda_{ij} - d \log \lambda_{jj} = (1-\sigma)\left(d \log \tau_{ij} + d \log w_i \right)$$

And so we can write (note $\sum_{i=1}^{N} \lambda_{ij} = 1 \Rightarrow \sum_{i=1}^{N} d\lambda_{ij} = 0$):

$$d \log W_j = -\left(\frac{1}{1-\sigma} \right) \sum_{i=1}^{N} \lambda_{ij} \left(d \log \lambda_{ij} - d \log \lambda_{jj} \right) = -\left(\frac{1}{1-\sigma} \right) \left(\sum_{i=1}^{N} \lambda_{ij} d \log \lambda_{ij} - d \log \lambda_{jj} \sum_{i=1}^{N} \lambda_{ij} \right)$$

$$= -\left(\frac{1}{1-\sigma} \right) \left(\sum_{i=1}^{N} d\lambda_{ij} - d \log \lambda_{jj} \right) = \left(\frac{1}{1-\sigma} \right) d \log \lambda_{jj}$$

Changes in Welfare

Changes in logged welfare given by

$$d\log W_j = \left(\frac{1}{1-\sigma}\right) d\log \lambda_{ij}$$

Therefore integrating changes means

$$\log W_j' - \log W_j = \int_{\lambda_{jj}}^{\lambda_{jj}'} \frac{d\log W_j}{d\log \lambda} d\log \lambda = \int_{\lambda_{jj}}^{\lambda_{jj}'} \left(\frac{1}{1-\sigma}\right) d\log \lambda = \left(\frac{1}{1-\sigma}\right) \left[\log \lambda_{jj}' - \log \lambda_{jj}\right]$$

Which yields relative welfare as (let $\widehat{W}_j \coloneqq W_j'/W_j$ and $\hat{\lambda}_j \coloneqq \lambda_{jj}'/\lambda_{jj}$):

$$\widehat{W}_j = \left(\hat{\lambda}_{jj}\right)^{\frac{1}{\epsilon}}$$

Which depends only on the trade elasticity ($\epsilon = 1 - \sigma$) and change in domestic expenditure share

Generalized Framework

ACR show that the previously derived expression holds in a wide class of models.

The model ingredients required are:

- **CES preferences** as we need the corresponding CES price indice for country j

$$P_j = \left(\int_{\omega \in \Omega_j} p_j(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}$$

- **Constant marginal costs** (linear production technologies) so that exporting doesn't affect domestic production costs except through general equilibrium effects

$$c_{ij}(\omega) = \tau_{ij} w_i \alpha_{ij}(\omega)$$

$$f_{ij} = w_i \xi_{ij}$$

where $\alpha_{ij}(\omega)$ is cost of producing good ω, and f_{ij} is the fixed cost to produce good ω in i for j

Generalized Framework

- **Perfect Competition or Monopolistic Competition** we want a constant markup over marginal cost. Assume no fixed costs if perfect competition. Can do either fixed set of producers or free entry for monopolistic competition. If free entry, assume constant fixed cost $w_i F_i$ to draw productivity.

- **Trade is Balanced** want expenditures to sum to income when working with expenditure shares

$$\sum_{i=1}^{N} X_{ij} = \sum_{i=1}^{N} X_{ji}, \qquad \forall j$$

- **Aggregate Profits are a Constant share of Revenues** related to the perfect competition and monopolistic competition assumption above, as those tend to generate this. Limits the margins through which trade affects income directly, so that welfare is captured by CES price index.

$$\pi_j / Y_j = \text{constant} \geq 0$$

Generalized Framework

- **CES import demand system** goes along with the CES preferences. Want changes in trade costs not to affect other countries' normalized trade shares, and a constant trade elasticity otherwise.

$$\frac{\partial \log\left(X_{ij}/X_{jj}\right)}{\partial \tau_{i'j}} = \begin{cases} 0, & \text{if } i \neq i' \\ \epsilon, & \text{if } i = i' \end{cases}, \qquad \forall i, i' \neq j$$

Note that under perfect competition, the above implies complete specialization.

Foreign Shocks

- A **foreign shock** in country j is a shock that affects labor endowments $\boldsymbol{L} \to \boldsymbol{L}'$, entry costs, $\boldsymbol{F} \to \boldsymbol{F}'$, and fixed costs, $\boldsymbol{\xi} \to \boldsymbol{\xi}'$, in countries other than j, as well as trade costs $\boldsymbol{\tau} \to \boldsymbol{\tau}'$ other than τ_{jj}. That is $L_j = L_j'$, $F_j = F_j'$, $\xi_{jj} = \xi_{jj}'$. $\boldsymbol{L} = \{L_i\}_i$, $\boldsymbol{\tau} = \{\tau_{ij}\}_{i,j}$ and similarly for other variables.

Proposition 1: Given the model restrictions, the welfare impact of the foreign shock for country j is

$$\widehat{W}_j = \left(\hat{\lambda}_{jj}\right)^{\frac{1}{\epsilon}}$$

Which is the same expression we derived for the simplified Armington Framework. This tells us that if we observe changes in domestic expenditure share and know the trade elasticity, then we know the changes in welfare.

Note, this cannot be used to predict welfare gains from trade if we don't know λ_{jj}'.

Welfare Gains from Trade

A simple corollary of proposition 1 is that the welfare losses from moving to autarky is

$$\widehat{W}_j^A = \left(\lambda_{jj}\right)^{-1/\epsilon}$$

Or equivalently, the gains from trade instead of autarky are $\left(\widehat{W}_j^A\right)^{-1}$. This follows from $\lambda_{jj}^A = 1$

Therefore, given observed domestic expenditure shares, and an estimated trade elasticity, the gains from trade are the same in all the models we considered (Armington, EK, Krugman, Melitz)

Overview of Proof: Perfect Competition

Basic steps of proof for models with perfect competition. Full proof see paper.

Step 1 Small changes in real income satisfy

$$d \log W_j = -d \log P_j$$

Step 2 Substituting prices into the price index

$$d \log P_j = \sum_{i=1}^{N} \lambda_{ij} d \log c_{ij}$$

Step 3 Plugging in trade elasticities

$$d \log P_j = \sum_{i=1}^{N} \lambda_{ij} \left(\frac{d \log \lambda_{ij} - d \log \lambda_{jj}}{\epsilon} \right)$$

Note that this involves both intensive and extensive margin changes and includes many substeps

Overview of Proof: Perfect Competition

Step 4 Noting that the trade share sum to 1 so that $\sum_{i=1}^{N} \lambda_i \, d \log \lambda_{ij} = 0$, yields

$$d \log P_j = -\frac{1}{\epsilon}\left(d \log \lambda_{jj}\right)$$

Step 5 Substitute in for income and integrating yields proposition 1

$$\widehat{W}_j = \frac{W_j'}{W_j} = \left(\frac{\lambda_{jj}'}{\lambda_{jj}}\right)^{1/\epsilon} = \hat{\lambda}_{jj}^{1/\epsilon}$$

Overview of Proof: Monopolistic Competition

For full proof see paper or [Treb Allen's lecture notes]()

Step 1 Small changes in real income satisfy

$$d \log W_j = -d \log P_j$$

Step 2 Small changes in the price index satisfy

$$d \log P_j = \sum_{i=1}^{N} \lambda_{ij} \left(\overbrace{d \log \tau_{ij} w_i}^{\text{intensive margin}} + \frac{\overbrace{d \log N_i + \gamma_{ij} d \log a_{ij}^*}^{\text{extensive margin}}}{1-\sigma} \right)$$

Where N_i is the number of firms that produce, a_{ij}^* is i's cutoff cost for serving country j, and

$$\gamma_{ij} := \frac{d \log \int_{a_{ij}^*}^{\infty} a^{\sigma-1} dG_i(a)}{d \log a_{ij}^*}$$

is the extensive margin elasticity

Overview of Proof: Monopolistic Competition

Step 4 Note the cutoff productivity satisfies zero profit condition

$$a_{ij}^* = \underbrace{\left(\frac{1}{\sigma}\right)^{\frac{1}{1-\sigma}}}_{\text{markup}} \underbrace{\left(\frac{Y_j}{\xi_{ij} w_i}\right)^{\frac{1}{1-\sigma}}}_{\text{fixed costs}} \underbrace{\left(\frac{\sigma}{\sigma - 1}\tau_{ij} w_i\right)}_{\text{common marginal costs}} \underbrace{P_j^{-1}}_{\text{price index}}$$

Step 5 Plugging in cutoffs to get changes in relative expenditure shares

$$d\log\lambda_{ij} - d\log\lambda_{jj} = \underbrace{d\log N_i - d\log N_j}_{\text{entrants}} + \underbrace{\left(1 - \sigma + \gamma_{ij}\right)\left(d\log\tau_{ij} w_i\right)}_{\text{average marginal costs}} - \underbrace{\frac{\gamma_{ij}}{1-\sigma}\left(d\log\xi_{ij} - d\log w_i\right)}_{\text{production fixed costs}}$$

Step 6 Plugging both of above into price index of step 2 yields

$$d\log P_j = \frac{1}{1 - \sigma + \sum_{i=1}^{N} \lambda_{ij}\gamma_{ij}}\left(-d\log\lambda_{jj} + \sum_{i=1}^{N} \lambda_{ij} d\log N_j\right)$$

Overview of Proof: Monopolistic Competition

Step 7 Note that number of producers is constant as profits are a constant share of revenue, θ,

$$\pi_j = N_j w_j F_j \Rightarrow N_j = \left(\frac{\pi_j}{w_j F_j}\right) = \left(\frac{\theta Y_j}{w_j F_j}\right) = \left(\frac{\theta w_j L_j}{w_j F_j}\right) = \frac{\theta L_j}{F_j}$$

Therefore $d \log N_j = 0$

Step 8 Note that the aggregate trade elasticity implies that

$$1 - \sigma + \sum_{i=1}^{N} \lambda_{ij} \gamma_{ij} = \epsilon$$

Step 9 Plugging in above two steps into step 6 yields

$$d \log P_j = -\frac{1}{\epsilon}\left(d \log \lambda_{jj}\right)$$

Overview of Proof: Monopolistic Competition

Step 10 Plugging price index into Welfare function

$$d \log W_j = \frac{1}{\epsilon}\left(d \log \lambda_{jj}\right)$$

and integrating yields proposition 1

$$\widehat{W}_j = \hat{\lambda}_{jj}^{1/\epsilon}$$

Conclusion

ACR show that:

- given a trade elasticity
- given observed domestic expenditure shares

⇒ welfare gains from trade same in wide class of commonly used models

Trade Elasticity Estimates

Simonovska and Waugh (2014) show that different models imply different trade elasticities at the micro level

- Therefore micro-level trade models quantitatively important for estimating gains from trade

Table 7: Macro and Micro Estimates of θ

			Macro + Micro	
Model	Macro	Micro	Identity Matrix	Optimal Matrix
Armington/Krugman	**4.63** [5.21, 4.09]	**5.24** [5.02, 5.45]	**5.24** [5.01, 5.45]	**5.23** [5.01, 5.44]
EK	**4.63** [5.21, 4.09]	**4.17** [4.00, 4.34]	**4.17** [4.03, 4.39]	**4.17** [4.04, 4.39]
Melitz	**4.63** [5.21, 4.09]	**3.69** [3.32, 4.21]	**3.70** [3.32, 4.12]	**3.70** [3.32, 4.12]
BEJK, ρ=1.37	**4.63** [5.21, 4.09]	**2.74** [2.62, 2.89]	**2.74** [2.62, 2.87]	**2.74** [2.63, 2.87]

Note: Macro presents estimates from (31). Micro presents estimates from (14). Macro + Micro presents estimates from (33). (Values within brackets report 90th and 10th confidence intervals.

Trade Elasticity Estimates

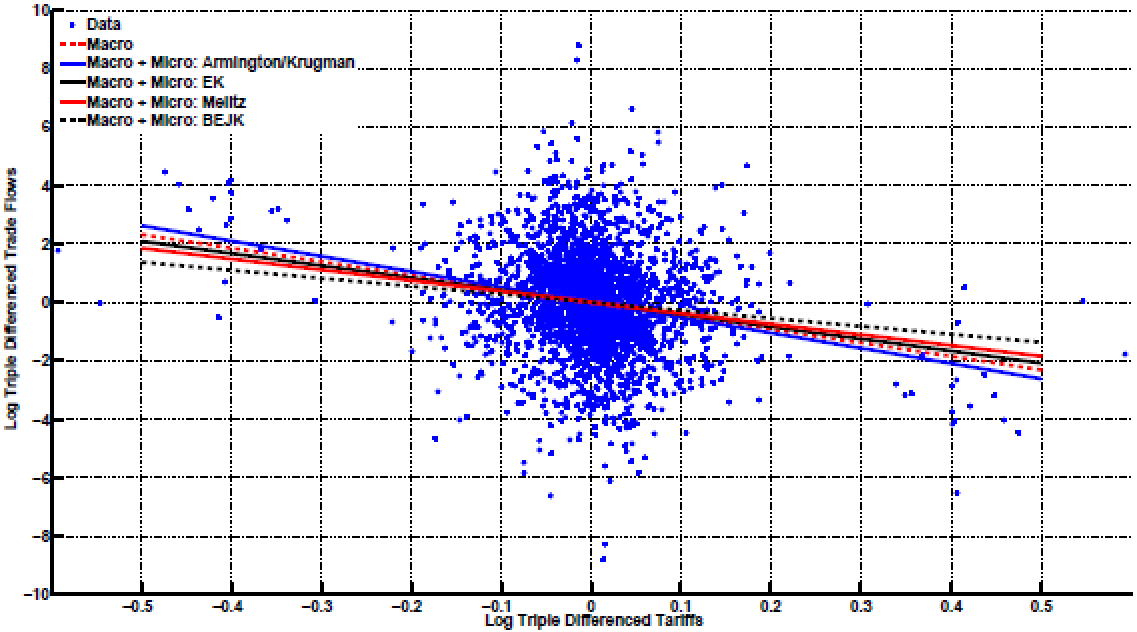

Figure 2: Trade Flows and Tariffs

Simonovska and Waugh (2014)