ECO 445/545, Spring 2016 **Professor Jack Rossbach**

Derivations for the Model in Problem Set 1, Question 2

Symmetric Equilibrium Solution on Last Two Pages

Consumers Problem:

Taking as given prices and wages, consumers in country $i = H, F$ maximize their utility

$$U^i\left(c_{1H}^i, c_{2H}^i, c_{1F}^i, c_{2F}^i\right) = \theta_1 \log\left(\left(c_{1H}^i\right)^\rho + \left(c_{1F}^i\right)^\rho\right) + \theta_2 \log\left(\left(c_{2H}^i\right)^\rho + \left(c_{2H}^i\right)^\rho\right)$$

Subject to their budget constraint

$$\sum_{m=1}^{2}\sum_{j=1}^{2} p_{mj}^i c_{mj}^i = w^i L^i$$

The Lagrangian is

$$\mathcal{L} = \theta_1 \log\left(\left(c_{1H}^i\right)^\rho + \left(c_{1F}^i\right)^\rho\right) + \theta_2 \log\left(\left(c_{2H}^i\right)^\rho + \left(c_{2H}^i\right)^\rho\right) + \lambda\left(w^i L^i - \sum_{m=1}^{2}\sum_{j=1}^{2} p_{mj}^i c_{mj}^i\right)$$

The FOC for good c_{mj}^i is (use the chain rule)

$$\left[c_{mj}^i\right]: \frac{\partial \mathcal{L}}{\partial c_{mj}^i} = \frac{\theta_m}{\left(c_{mH}^i\right)^\rho + \left(c_{mF}^i\right)^\rho}\rho\left(c_{mj}^i\right)^{\rho-1} - \lambda p_{mj}^i = 0$$

Combining the above FOCs for both varieties of good m, we have

$$\frac{\frac{\theta_m}{\left(c_{mH}^i\right)^\rho + \left(c_{mF}^i\right)^\rho}\rho\left(c_{mH}^i\right)^{\rho-1}}{\frac{\theta_m}{\left(c_{mH}^i\right)^\rho + \left(c_{mF}^i\right)^\rho}\rho\left(c_{mF}^i\right)^{\rho-1}} = \frac{\lambda p_{mH}^i}{\lambda p_{mF}^i}$$

Simplifying

$$\frac{\left(c_{mH}^i\right)^{\rho-1}}{\left(c_{mF}^i\right)^{\rho-1}} = \frac{p_{mH}^i}{p_{mF}^i}$$

Rearranging and raising both sides to $\frac{1}{\rho-1}$ power

$$c_{mH}^i = c_{mF}^i\left(\frac{p_{mH}^i}{p_{mF}^i}\right)^{\frac{1}{\rho-1}}$$

Noting that $\rho < 1$, we flip the prices (and raise to the power -1) to get relative consumption for varieties of the same good.

$$c_{mH}^{i}=c_{mF}^{i}\left(\frac{p_{mF}^{i}}{p_{mH}^{i}}\right)^{\frac{1}{1-\rho}}, \qquad (2)$$

This means that if the price of one variety increases, you purchase more of the other variety. Note also that this means we have

$$\left(c_{mH}^{i}\right)^{\rho}+\left(c_{mF}^{i}\right)^{\rho}=\left(c_{mF}^{i}\left(\frac{p_{mF}^{i}}{p_{mH}^{i}}\right)^{\frac{1}{1-\rho}}\right)^{\rho}+\left(c_{mF}^{i}\right)^{\rho}=\left(c_{mF}^{i}\right)^{\rho}\left(1+\left(\frac{p_{mF}^{i}}{p_{mH}^{i}}\right)^{\frac{\rho}{1-\rho}}\right)$$

Which gives

$$\left(c_{mH}^{i}\right)^{\rho}+\left(c_{mF}^{i}\right)^{\rho}=\left(c_{mF}^{i}\right)^{\rho}\left(\frac{\left(p_{mH}^{i}\right)^{\frac{\rho}{1-\rho}}+\left(p_{mF}^{i}\right)^{\frac{\rho}{1-\rho}}}{\left(p_{mH}^{i}\right)^{\frac{\rho}{1-\rho}}}\right)$$

Going back to (1) and now using it for different goods (rather than different varieties) we have

$$\frac{\frac{\theta_1}{\left(c_{1H}^{i}\right)^{\rho}+\left(c_{1F}^{i}\right)^{\rho}}\rho\left(c_{1F}^{i}\right)^{\rho-1}}{\frac{\theta_2}{\left(c_{2H}^{i}\right)^{\rho}+\left(c_{2F}^{i}\right)^{\rho}}\rho\left(c_{2F}^{i}\right)^{\rho-1}}=\frac{\lambda p_{1F}^{i}}{\lambda p_{2F}^{i}}$$

Cancelling the λ's and plugging in our expression for $\left(c_{mH}^{i}\right)^{\rho}+\left(c_{mF}^{i}\right)^{\rho}$ we get

$$\frac{\frac{\theta_1}{\left(c_{1F}^{i}\right)^{\rho}\left(\frac{\left(p_{1H}^{i}\right)^{\frac{\rho}{1-\rho}}+\left(p_{1F}^{i}\right)^{\frac{\rho}{1-\rho}}}{\left(p_{1H}^{i}\right)^{\frac{\rho}{1-\rho}}}\right)}\rho\left(c_{1F}^{i}\right)^{\rho-1}}{\frac{\theta_2}{\left(c_{2F}^{i}\right)^{\rho}\left(\frac{\left(p_{2H}^{i}\right)^{\frac{\rho}{1-\rho}}+\left(p_{2F}^{i}\right)^{\frac{\rho}{1-\rho}}}{\left(p_{2H}^{i}\right)^{\frac{\rho}{1-\rho}}}\right)}\rho\left(c_{2F}^{i}\right)^{\rho-1}}=\frac{p_{1F}^{i}}{p_{2F}^{i}}$$

Simplifying and rearranging so expenditures of good 2 are alone on the RHS gives

$$p_{2F}^{i}c_{2F}^{i}=\frac{\theta_2}{\theta_1}p_{1F}^{i}c_{1F}^{i}\frac{\left(\frac{\left(p_{1H}^{i}\right)^{\frac{\rho}{1-\rho}}+\left(p_{1F}^{i}\right)^{\frac{\rho}{1-\rho}}}{\left(p_{1H}^{i}\right)^{\frac{\rho}{1-\rho}}}\right)}{\left(\frac{\left(p_{2H}^{i}\right)^{\frac{\rho}{1-\rho}}+\left(p_{2F}^{i}\right)^{\frac{\rho}{1-\rho}}}{\left(p_{2H}^{i}\right)^{\frac{\rho}{1-\rho}}}\right)}, \qquad (3)$$

Now, going back to (2) and plugging (2) into the budget constraint we have

$$p_{1H}^{i}\left(c_{1F}^{i}\left(\frac{p_{1F}^{i}}{p_{1H}^{i}}\right)^{\frac{1}{1-\rho}}\right)+p_{1F}^{i}c_{1F}^{i}+p_{2H}^{i}\left(c_{2F}^{i}\left(\frac{p_{2F}^{i}}{p_{2H}^{i}}\right)^{\frac{1}{1-\rho}}\right)+p_{2F}^{i}c_{2F}^{i}=w^{i}L^{i}$$

Collecting terms

$$p_{1F}^{i}c_{1F}^{i}\left(1+\left(\frac{p_{1F}^{i}}{p_{1H}^{i}}\right)^{\frac{1}{1-\rho}-1}\right)+p_{2F}^{i}c_{2F}^{i}\left(1+\left(\frac{p_{2F}^{i}}{p_{2H}^{i}}\right)^{\frac{1}{1-\rho}-1}\right)=w^{i}L^{i}$$

Simplifying

$$p_{1F}^{i}c_{1F}^{i}\left(\frac{\left(p_{1H}^{i}\right)^{\frac{\rho}{1-\rho}}+\left(p_{1F}^{i}\right)^{\frac{\rho}{1-\rho}}}{\left(p_{1H}^{i}\right)^{\frac{\rho}{1-\rho}}}\right)+p_{2F}^{i}c_{2F}^{i}\left(\frac{\left(p_{2H}^{i}\right)^{\frac{\rho}{1-\rho}}+\left(p_{2F}^{i}\right)^{\frac{\rho}{1-\rho}}}{\left(p_{2H}^{i}\right)^{\frac{\rho}{1-\rho}}}\right)=w^{i}L^{i}$$

Plugging (3) into the above

$$p_{1F}^{i}c_{1F}^{i}\left(\frac{\left(p_{1H}^{i}\right)^{\frac{\rho}{1-\rho}}+\left(p_{1F}^{i}\right)^{\frac{\rho}{1-\rho}}}{\left(p_{1H}^{i}\right)^{\frac{\rho}{1-\rho}}}\right)+\left[\frac{\theta_2}{\theta_1}p_{1F}^{i}c_{1F}^{i}\frac{\left(\frac{\left(p_{1H}^{i}\right)^{\frac{\rho}{1-\rho}}+\left(p_{1F}^{i}\right)^{\frac{\rho}{1-\rho}}}{\left(p_{1H}^{i}\right)^{\frac{\rho}{1-\rho}}}\right)}{\left(\frac{\left(p_{2H}^{i}\right)^{\frac{\rho}{1-\rho}}+\left(p_{2F}^{i}\right)^{\frac{\rho}{1-\rho}}}{\left(p_{2H}^{i}\right)^{\frac{\rho}{1-\rho}}}\right)}\right]\left(\frac{\left(p_{2H}^{i}\right)^{\frac{\rho}{1-\rho}}+\left(p_{2F}^{i}\right)^{\frac{\rho}{1-\rho}}}{\left(p_{2H}^{i}\right)^{\frac{\rho}{1-\rho}}}\right)$$
$$=w^{i}L^{i}$$

Simplifying by cancelling terms

$$p_{1F}^{i}c_{1F}^{i}\left(\frac{\left(p_{1H}^{i}\right)^{\frac{\rho}{1-\rho}}+\left(p_{1F}^{i}\right)^{\frac{\rho}{1-\rho}}}{\left(p_{1H}^{i}\right)^{\frac{\rho}{1-\rho}}}\right)+\frac{\theta_2}{\theta_1}p_{1F}^{i}c_{1F}^{i}\left(\frac{\left(p_{1H}^{i}\right)^{\frac{\rho}{1-\rho}}+\left(p_{1F}^{i}\right)^{\frac{\rho}{1-\rho}}}{\left(p_{1H}^{i}\right)^{\frac{\rho}{1-\rho}}}\right)=w^{i}L^{i}$$

Collecting terms

$$p_{1F}^{i}c_{1F}^{i}\left(\frac{\left(p_{1H}^{i}\right)^{\frac{\rho}{1-\rho}}+\left(p_{1F}^{i}\right)^{\frac{\rho}{1-\rho}}}{\left(p_{1H}^{i}\right)^{\frac{\rho}{1-\rho}}}\right)\left(1+\frac{\theta_2}{\theta_1}\right)=w^{i}L^{i}$$

$$p_{1F}^{i}c_{1F}^{i}\left(\frac{\left(p_{1H}^{i}\right)^{\frac{\rho}{1-\rho}}+\left(p_{1F}^{i}\right)^{\frac{\rho}{1-\rho}}}{\left(p_{1H}^{i}\right)^{\frac{\rho}{1-\rho}}}\right)\left(\frac{\theta_1+\theta_2}{\theta_1}\right)=w^{i}L^{i}$$

Rearranging to have consumption alone on the LHS gives

$$c_{1F}^{i}=\left(\frac{\theta_1}{\theta_1+\theta_2}\right)\left(\frac{\left(p_{1H}^{i}\right)^{\frac{\rho}{1-\rho}}}{p_{1F}^{i}}\right)\frac{w^{i}L^{i}}{\left(\left(p_{1H}^{i}\right)^{\frac{\rho}{1-\rho}}+\left(p_{1F}^{i}\right)^{\frac{\rho}{1-\rho}}\right)}$$

Which gives consumption for the foreign variety of good 1. We can use the above (2) and (3) to get

$$c_{mj}^{i}=\left(\frac{\theta_m}{\theta_1+\theta_2}\right)\left(\frac{\left(p_{1(-j)}^{i}\right)^{\frac{\rho}{1-\rho}}}{p_{1j}^{i}}\right)\frac{w^{i}L^{i}}{\left(\left(p_{mH}^{i}\right)^{\frac{\rho}{1-\rho}}+\left(p_{mF}^{i}\right)^{\frac{\rho}{1-\rho}}\right)},\qquad (4)$$

Where $(-j)$ stands for "not j". That is, if $j = H$ then $(-j) = F$. This gives us an expression for the consumption of each good, in terms of prices and income. We are going to write it in a different format however.

Note that (4) implies that

$$p_{mF}^i c_{mF}^i + p_{mH}^i c_{mH}^i = \left(\frac{\theta_m}{\theta_1 + \theta_2}\right)\left((p_{mH}^i)^{\frac{\rho}{1-\rho}}\right)\frac{w^i L^i}{\left((p_{1H}^i)^{\frac{\rho}{1-\rho}} + (p_{1F}^i)^{\frac{\rho}{1-\rho}}\right)} + \left(\frac{\theta_m}{\theta_1 + \theta_2}\right)\left((p_{1F}^i)^{\frac{\rho}{1-\rho}}\right)\frac{w^i L^i}{\left((p_{1H}^i)^{\frac{\rho}{1-\rho}} + (p_{1F}^i)^{\frac{\rho}{1-\rho}}\right)}$$

Combining Terms

$$p_{mF}^i c_{mF}^i + p_{mH}^i c_{mH}^i = \left(\frac{\theta_m}{\theta_1 + \theta_2}\right)\left(\frac{(p_{1H}^i)^{\frac{\rho}{1-\rho}} + (p_{1F}^i)^{\frac{\rho}{1-\rho}}}{(p_{1H}^i)^{\frac{\rho}{1-\rho}} + (p_{1F}^i)^{\frac{\rho}{1-\rho}}}\right) w^i L^i$$

Simplifying

$$p_{mF}^i c_{mF}^i + p_{mH}^i c_{mH}^i = \left(\frac{\theta_m}{\theta_1 + \theta_2}\right) w^i L^i, \qquad (5)$$

Which means that consumers still spend a constant fraction of their income on each good (spending on the good is the sum of spending on each variety of that good). This is exactly as expected since we have Cobb-Douglas preferences across goods.

Combining (5) with (2) gives

$$p_{mH}^i \left(c_{1F}^i \left(\frac{p_{mF}^i}{p_{mH}^i}\right)^{\frac{1}{1-\rho}} \right) + p_{mF}^i c_{mF}^i = \left(\frac{\theta_m}{\theta_1 + \theta_2}\right) w^i L^i$$

Combining terms

$$(p_{mF}^i)^{\frac{1}{1-\rho}} c_{mF}^i (p_{mH}^i)^{\frac{-\rho}{1-\rho}} + p_{mF}^i c_{mF}^i = \left(\frac{\theta_m}{\theta_1 + \theta_2}\right) w^i L^i$$

Separating p_{mF}^i into $(p_{mF}^i)^{\frac{-\rho}{1-\rho}}(p_{mF}^i)^{\frac{1}{1-\rho}}$

$$(p_{mF}^i)^{\frac{1}{1-\rho}} c_{mF}^i (p_{mH}^i)^{\frac{-\rho}{1-\rho}} + (p_{mF}^i)^{\frac{-\rho}{1-\rho}} (p_{mF}^i)^{\frac{1}{1-\rho}} c_{mF}^i = \left(\frac{\theta_m}{\theta_1 + \theta_2}\right) w^i L^i$$

Combining terms

$$\left(p_{mF}^{i}\right)^{\frac{1}{1-\rho}} c_{mF}^{i}\left(\left(p_{mH}^{i}\right)^{\frac{-\rho}{1-\rho}}+\left(p_{mF}^{i}\right)^{\frac{-\rho}{1-\rho}}\right)=\left(\frac{\theta_m}{\theta_1+\theta_2}\right) w^i L^i$$

Rearranging so that consumption is alone on the RHS gives

$$c_{mF}^{i}=\left(\frac{\theta_m}{\theta_1+\theta_2}\right)\frac{w^i L^i}{\left(p_{mF}^{i}\right)^{\frac{1}{1-\rho}}\left(\left(p_{mH}^{i}\right)^{\frac{-\rho}{1-\rho}}+\left(p_{mF}^{i}\right)^{\frac{-\rho}{1-\rho}}\right)}, (6)$$

Note that this is the same as (4), just written a different way. The reason we write it this way is because we can define it using a Price Index that will be useful for welfare analysis later on in the semester.

Using (6) we can write consumption as

$$c_{mj}^{i}=\frac{\left(\frac{\theta_m}{\theta_1+\theta_2}\right) w^i L^i}{\left(p_{mj}^{i}\right)^{\frac{1}{1-\rho}}\left(P_m^i\right)^{-\left(\frac{\rho}{1-\rho}\right)}}, \qquad (C1)$$

Where we define the price index for good m in country i as

$$P_m^i \equiv \left(\left(p_{mH}^{i}\right)^{\frac{-\rho}{1-\rho}}+\left(p_{mF}^{i}\right)^{\frac{-\rho}{1-\rho}}\right)^{-\left(\frac{1-\rho}{\rho}\right)}, \qquad (C2)$$

Which gives us the solution to the consumer's problem $\forall i, j=H, F; m=1{,}2$.

Firms Problem

The firm's problem is to maximize profits subject to its production function. Firms are perfectly competitive, so they take prices as given. Since we have constant labor input costs, we can have firms maximize profits separately for both the domestic and export markets.

Domestic Market

For the domestic market firm m located in j solves

$$\max_{\left\{\mathrm{y}_{\mathrm{mj}}^{\mathrm{j}}, l_{mj}^{j}\right\}} p_{mj}^{j} y_{mj}^{j}-w^j l_{mj}^{j}$$

Subject to their production function

$$y_{mj}^{j}=\frac{1}{a_{mj}} l_{mj}^{j}$$

Where a_{mj} is the firm's unit labor cost.

Substituting the production function into profits means that firms solve

$$\max_{\left\{l_{mj}^{j}\right\}} p_{mj}^{j}\left(\frac{1}{a_{mj}} l_{mj}^{j}\right)-w^j l_{mj}^{j}$$

The FOC with respect to labor is given by

$$\left[l_{mj}^{j}\right]:\ p_{mj}^{j}\frac{1}{a_{mj}}-w^{j}=0$$

Which means that domestic prices satisfy

$$p_{mj}^{j}=a_{mj}w^{j},\qquad if\ l_{mj}^{j}>0,$$

Note that in this economy (unlikely the standard 2x2 Ricardian model), we will always have $l_{mj}^{j}>0$ since consumers want to consume some of each variety regardless of the price, which we know from (C1).

Foreign Market

For the domestic market firm m located in j solves (note $(-j)$ means not j, so if $j=H$ then $(-j)=F$)

$$\max_{\left\{\mathrm{y}_{\mathrm{mj}}^{\mathrm{j}},l_{mj}^{j}\right\}} p_{mj}^{(-j)}y_{mj}^{(-j)}-w^{j}l_{mj}^{(-j)}$$

Subject to the production function

$$y_{mj}^{(-j)}=\frac{1}{\tau a_{mj}}l_{mj}^{(-j)}$$

Where a_{mj} is the firm's unit labor cost and τ is the iceburg cost to export one unit of output outside of the country. Since we have constant labor input costs, we internalize the iceburg cost directly into the firms production function. The production function can be read as, to deliver one unit of output to the foreign market, requires τa_{mj} units of labor.

Substituting the production function into profits means that firms solve

$$\max_{\left\{l_{mj}^{j}\right\}} p_{mj}^{(-j)}\left(\frac{1}{\tau a_{mj}}l_{mj}^{(-j)}\right)-w^{j}l_{mj}^{(-j)}$$

The FOC with respect to labor is given by

$$\left[l_{mj}^{j}\right]:\ p_{mj}^{(-j)}\frac{1}{a_{mj}}-w^{j}=0$$

Which means that domestic prices satisfy

$$p_{mj}^{(-j)}=\tau a_{mj}w^{j},\qquad if\ l_{mj}^{(-j)}>0$$

Note that, as long as trade costs are finite, we will always have $l_{mj}^{(-j)}>0$ in equilibrium.

Combined

First, define $\tau_{j}^{i}\equiv\begin{cases}1, & if\ i=j\\ \tau, & if\ i\neq j\end{cases}$ as it is in the problem.

Then we can write the solutions to both the domestic and foreign markets jointly as (for this model we will leave out the condition that $l_{mj}^{i} > 0$, since we know it always will be in equilibrium)

$$p_{mj}^{i} = \tau_j^i a_{mj} w^j, \qquad (F1)$$

And the production function is given by,

$$y_{mj}^{i} = \frac{1}{\tau_j^i a_{mj}} l_{mj}^{i}, \qquad (F2)$$

Both of which hold $\forall i, j = H, F; m = 1,2$.

Market Clearing

The last part of the problem is market clearing conditions. We have market clearing hold separately for each destination now, so the goods market clearing is, $\forall i, j = H, F; m = 1,2$

$$c_{mj}^{i} = y_{mj}^{i}, \qquad (M1)$$

While the labor market clearing condition holds for each country, $j = H, F$

$$l_{1j}^{H} + l_{1j}^{F} + l_{2j}^{H} + l_{2j}^{F} = L^j, \qquad (M2)$$

Solving the Symmetric Equilibrium

The "symmetric equilibrium" can either mean that countries are identical or mirrored. In this case, I'll just assume that both countries are exactly the same. Solving the symmetric equilibrium in this case is easy.

I will solve for the equilibrium where $L^H = L^F = L$, and $a_{1H} = a_{1F} = a_1$; $a_{2H} = a_{2F} = a_2$. Since countries are exactly the same, we know that prices for the domestic and exported good must be the same in each country and wages must be the same in each country.

Step 1: Normalize a price

I normalize $w^H = 1$. Note since countries are identical, this means $w^F = 1$ as well (you can verify this by solving the equilibrium without imposing symmetry and getting an expression for relative wages, then plugging in symmetry afterwards).

Step 2: Plug wages into prices

Plugging $w^j = 1$ into (F1) (along with $a_{mj} = a_m$ due to symmetry) gives

$$p_{mj}^{i} = \tau_j^i a_m$$

Step 3: Plug prices and wages into consumption

Plugging (C2) into (C1) to get a version of (6), then plugging wages and prices (along with $L^i = L$) gives consumption

$$c_{mj}^{i} = \left(\frac{\theta_m}{\theta_1 + \theta_2}\right) \frac{L}{\left(\tau_j^i a_m\right)^{\frac{1}{1-\rho}} \left(\left(\tau_H^i a_m\right)^{\frac{-\rho}{1-\rho}} + \left(\tau_F^i a_m\right)^{\frac{-\rho}{1-\rho}}\right)}$$

Step 4: Use production function and market clearing to find labor and output allocations

From the above we have consumption allocations, and we can plug that into (M1) to get output

$$y_{mj}^{i} = \left(\frac{\theta_m}{\theta_1 + \theta_2}\right) \frac{L}{\left(\tau_j^i a_m\right)^{\frac{1}{1-\rho}} \left(\left(\tau_H^i a_m\right)^{\frac{-\rho}{1-\rho}} + \left(\tau_F^i a_m\right)^{\frac{-\rho}{1-\rho}}\right)}$$

And we can plug that into (F2) to get labor [the exponent on $\tau_j^i a_m$ changed to $\frac{\rho}{1-\rho}$

$$l_{mj}^{i} = \left(\frac{\theta_m}{\theta_1 + \theta_2}\right) \frac{L}{\left(\tau_j^i a_m\right)^{\frac{\rho}{1-\rho}} \left(\left(\tau_H^i a_m\right)^{\frac{-\rho}{1-\rho}} + \left(\tau_F^i a_m\right)^{\frac{-\rho}{1-\rho}}\right)}$$

That's it. We have our symmetric equilibrium then.

Symmetric Free Trade Equilibrium

We can go one step further and impose free trade $\tau = 1$, in which case we have

$$\left(\tau_j^i a_m\right)^{\frac{1}{1-\rho}} \left(\left(\tau_H^i a_m\right)^{\frac{-\rho}{1-\rho}} + \left(\tau_F^i a_m\right)^{\frac{-\rho}{1-\rho}}\right) = (a_m)^{\frac{1}{1-\rho}} \left((a_m)^{\frac{-\rho}{1-\rho}} + (a_m)^{\frac{-\rho}{1-\rho}}\right) =$$

$$= (a_m)^{\frac{-\rho}{1-\rho} + \frac{1}{1-\rho}} (1+1) = (a_m)^1 (2) = 2a_m$$

Therefore consumption and output become

$$c_{mj}^{i} = y_{mj}^{i} = \left(\frac{\theta_m}{\theta_1 + \theta_2}\right) \frac{L}{2a_m}$$

While labor input becomes

$$l_{mj}^{i} = \left(\frac{\theta_m}{\theta_1 + \theta_2}\right) \frac{L}{2}$$

So, in the symmetric equilibrium, labor input isn't affected by labor productivities, which is an artifact of our Cobb-Douglas preferences across goods, and the fact that each country only makes one variety for each good.

Plugging in free trade means prices are given by

$$p_{mj}^{i} = a_m$$

While again we normalize wages equal to 1

$$w^j = 1$$